

### **Compositional Protocol Logic**

### Outline

#### Floyd-Hoare logic of programs

- Compositional reasoning about properties of programs
- DDMP protocol logic
  - Developed by Datta, Derek, Mitchell, and Pavlovic for logical reasoning about security properties

## Floyd-Hoare Logic

#### Main idea: before-after assertions

- F <P> G
  - If F is true before executing P, then G is true after
- Total correctness or partial correctness
  - Total correctness: F [P] G
    - If F is true, then <u>P will halt</u> and G will be true
  - Partial correctness: F {P} G
    - If F is true and if P halts, then G will be true

### While Programs

where x is any variable e is any integer expression B is a Boolean expression (true or false)

### Assignment and Rule of Consequence

Assignment axiom:  $F(t) \{ x := t \} F(x)$ 

- If F holds for t, and t is assigned to x, then F holds for x aftewards
- This assumes that there is no aliasing!
- Examples:

7=7{ x := 7 }x=7(y+1)>0{ x := y+1 }x>0x+1=2{ x := x+1 }x=2

Rule of consequence:
If F { P } G and F'  $\rightarrow$  F and G  $\rightarrow$  G',
then F' { P } G'

### Simple Examples

 $\mathbf{X} =$ 

Assertion: x=1 { x := x+1 } x=2 Proof: x+1=2 { x := x+1 } x=2

### Conditional

F & B { P } G F & B { Q } G F { if B then P else Q } G

• Example:

true { if  $y \ge 0$  then x := y else x := -y }  $x \ge 0$ 

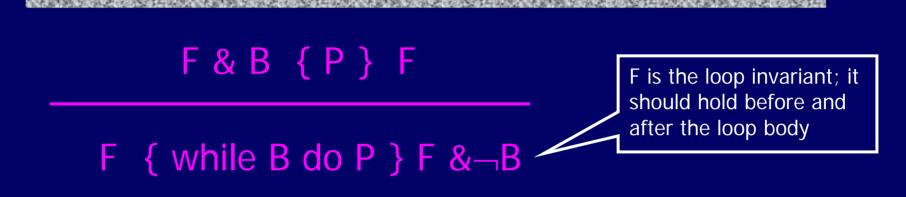


F { P } G G { Q } H F { P; Q } H

• Example:

 $x=0 \{ x := x+1 ; x := x+1 \} x=2$ 

### Loop Invariant



• Example:

true { while  $x \neq 0$  do x := x-1 } x=0

### Example: Compute d=x-y

Assertion:  $y \le x$  {d:=0; while (y+d) < x do d := d+1} y+d=x
P
B
Q
Proof:
• Choose loop invariant  $F = y+d \le x$   $y+d \le x \& B \ \{Q\} \ y+d \le x$   $y+d \le x \ \{while \ B \ do \ Q\} \ y+d \le x \& \neg B$ Assertion:  $y \le d \le x \& \neg (y+d \le x), thus \ y+d \le x \&$ 

- <u>Important</u>: proving a property of the entire loop has been reduced to proving a property of one iteration of the loop
- To prove y+d≤x & B {Q} y+d≤x, use assignment axiom and sequence rule

### **Goal: Logic for Security Protocols**

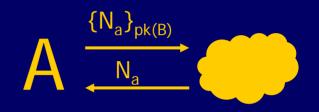
"Floyd-Hoare" reasoning about security properties

- Would like to derive <u>global</u> properties of protocols from <u>local</u> assertions about each protocol participant
- Use a rigorous logical framework to formalize the reasoning that each participant carries out
- Compositionality is important
  - Security properties must hold even if the protocol is executed in parallel with other protocols
  - Compositionality is the main advantage of process calculi and protocol logics

## Intuition

#### Reason about local information

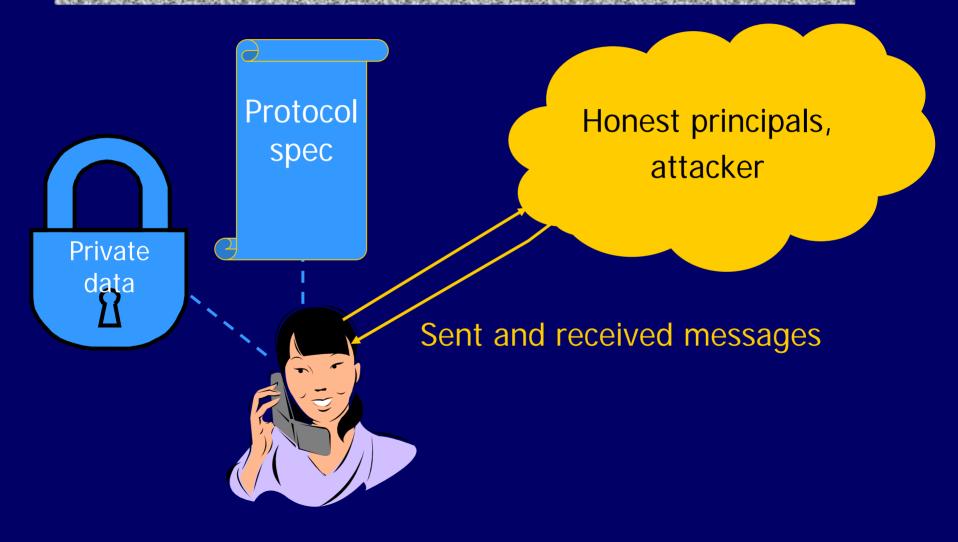
- I chose a fresh, unpredictable number
- I sent it out encrypted
- I received it decrypted
- Therefore: someone decrypted it



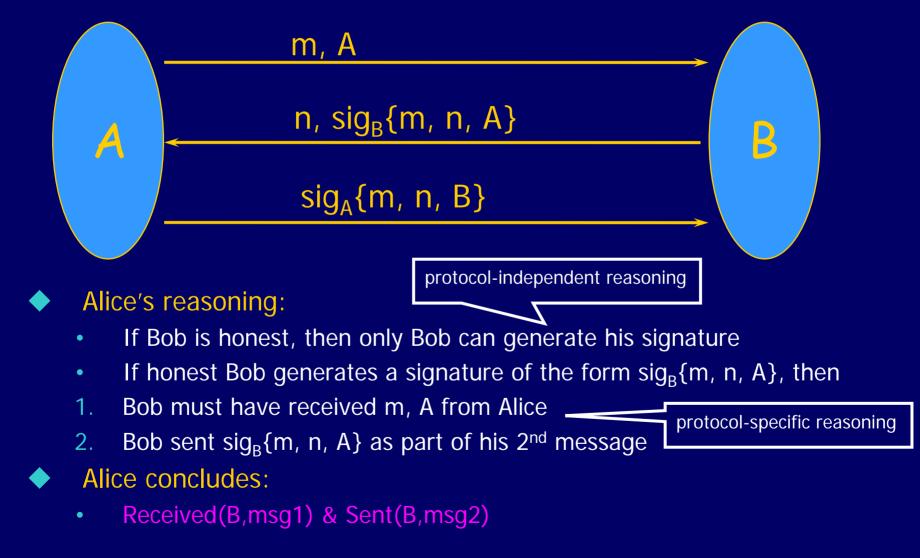
#### Incorporate knowledge about protocol into reasoning

- According to the protocol specification, server only sends m if it received m'
- If server not corrupt and I receive m signed by server, then server received m'

### Alice's "View" of the Protocol



### Example: Challenge-Response



## **Protocol Composition Logic**

[Datta et al.]

#### A formal language for describing protocols

- Terms and actions instead of informal arrows-andmessages notation
- Operational semantics
  - Describe how the protocol executes

#### Protocol logic

• State security properties (in particular, secrecy and authentication)

#### Proof system

 Axioms and inference rules for formally proving security properties

### Terms

constant variable name key tuple signature encryption

### Actions

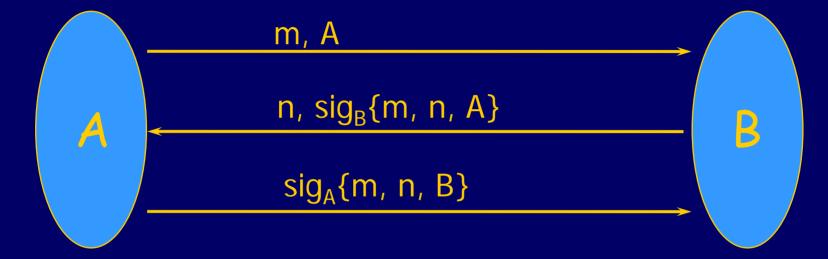
new mgeneratesend U, V, tsend terr

receive U, V, x match t/p(x) generate fresh value send term t from U to V receive term and assign into variable x match term t against pattern p(x)

#### ♦ A thread is a sequence of actions

- Defines the "program" for a protocol participant, i.e., what messages he sends and receives and the checks he performs
- For notational convenience, omit "match" actions
  - Write "receive  $sig_B{A, n}$ " instead of "receive x; match x/sig\_B{A, n}"

### **Challenge-Response Threads**



InitCR(A, X) = [

]

```
new m;
send A, X, {m, A};
receive X, A, {x, sig<sub>x</sub>{m, x, A}};
send A, X, sig<sub>A</sub>{m, x, X};
```

RespCR(B) = [
 receive Y, B, {y, Y};
 new n;
 send B, Y, {n, sig<sub>B</sub>{y, n, Y}};
 receive Y, B, sig<sub>Y</sub>{y, n, B};
]

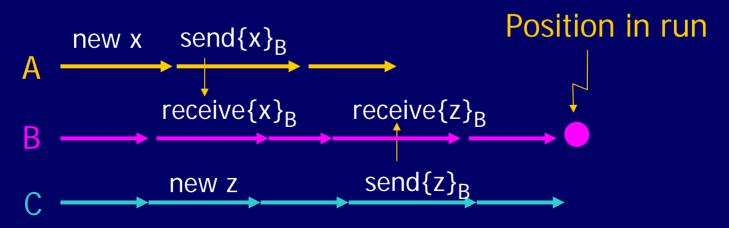
### **Execution Model**

#### A protocol is a finite set of roles

 Initial configuration specifies a set of principals and keys; assignment of ≥1 role to each principal

#### A run is a concurrent execution of the roles

- Models a protocol session
- Send and receive actions are matched up



### **Action Formulas**

 Predicates over action sequences
 a ::= Send(X,m) | Message m was sent in thread X Receive(X,m) | Message m was received in thread X New(X,t) | Term t was generated as new in X Decrypt(X,t) | Term t was decrypted in thread X Verify(X,t) | Term t was verified in X

### Formulas

 $\phi ::= a$ Has(X,m)

> Fresh(X,t) Honest(N)

Contains(t,t') |  $\neg \phi \mid \phi_1 \land \phi_2 \mid \exists x \phi \mid$   $\bigcirc \phi \mid \diamondsuit \phi$  $\phi was true$ 

Modal operator [actions]<sub>χ</sub> φ

#### Action formula

Thread X created m or received a message containing m and has keys to extract m from the message Term t hasn't been "seen" outside X Principal N follows protocol rules in all of its threads Term t contains subterm t'

Temporal logic operators on past actions

After actions, X reasons  $\phi$ 

### **Trace Semantics**

#### Protocol Q

- Defines a set of roles (e.g., initiator and responder)
- Run R
  - Sequence of actions by principals following protocol roles and the attacker (models a protocol session)

#### Satisfaction

- Q, R |=  $[actions]_P \varphi$ 
  - Some role of principal P in R performs exactly actions and  $\phi$  is true in the state obtained after actions complete
- $Q \models [actions]_P \varphi$

- Q, R |=  $[actions]_P \phi$  for all runs R of Q

### **Specifying Authentication**

### Initiator authentication in Challenge-Response

After initiator executes his program

If B is honest...

 $CR \models [InitCR(A, B)]_A Honest(B) \supset$ ActionsInOrder(

Send(A, {A,B,m}), Receive(B, {A,B,m}), Send(B, {B,A,{n, sig<sub>B</sub>{m, n, A}}}), Receive(A, {B,A,{n, sig<sub>B</sub>{m, n, A}}})

> ...then msg sends and receives must have happened in order prescribed by protocol spec

## **Specifying Secrecy**

#### Shared secret in key establishment

After initiator executes his program

If B is honest...

 $\begin{array}{lll} \mathsf{KE} \ & \mid = \ \left[ \ \mathsf{Init}\mathsf{KE}(\mathsf{A}, \ \mathsf{B}) \ \right]_{\mathsf{A}} \ \mathsf{Honest}(\mathsf{B}) \supset \\ & (\mathsf{Has}(\mathsf{X}, \ \mathsf{m}) \supset \mathsf{X} {=} \mathsf{A} \lor \mathsf{X} {=} \mathsf{B} \ ) \end{array}$ 

... then if some party X knows secret m, then X can only be either A, or B

### **Proof System**

 Goal: formally prove properties of security protocols

#### Axioms are simple formulas

- Provable by hand
- Inference rules are proof steps

Theorem is a formula obtained from axioms by application of inference rules

### Sample Axioms

#### New data

- [ new x ]<sub>P</sub> Has(P,x)
- [new x]<sub>P</sub> Has(Y,x)  $\supset$  Y=P
- Acquiring new knowledge
  - [ receive m ]<sub>P</sub> Has(P,m)
- Performing actions
  - [send m]<sub>P</sub> \$Send(P,m)
  - [match x/sig<sub>x</sub>{m}] P

# Reasoning About Cryptography

#### Pairing

- Has(X, {m,n})  $\supset$  Has(X, m)  $\land$  Has(X, n)
- Symmetric encryption
  - Has(X, enc<sub>K</sub>(m))  $\land$  Has(X, K<sup>-1</sup>)  $\supset$  Has(X, m)
- Public-key encryption
  - Honest(X)  $\land \diamondsuit$  Decrypt(Y, enc<sub>X</sub>{m})  $\supset X = Y$

#### Signatures

Honest(X) ∧ ◇Verify(Y, sig<sub>X</sub>{m}) ⊃
 ∃ m' (◇Send(X, m') ∧ Contains(m', sig<sub>X</sub>{m}))

### Sample Inference Rules

[ actions ]<sub>P</sub> Has(X, t) [ actions; action ]<sub>P</sub> Has(X, t)

 $\begin{bmatrix} actions \end{bmatrix}_{P} \phi \qquad [ actions ]_{P} \phi \\ \hline [ actions ]_{P} \phi \land \phi \end{bmatrix}$ 

### Honesty Rule

 $\forall \text{roles R of Q. } \forall \text{ initial segments A} \subseteq \text{R.}$  $\begin{array}{c} Q & |- & [ \text{ A } ]_X \phi \\ \hline Q & |- & \text{Honest}(X) \supset \phi \end{array}$ 

- Finitary rule (finite number of premises to choose from)
   Typical protocol has 2-3 roles typical role has 1-3 actions
  - Typical protocol has 2-3 roles, typical role has 1-3 actions
- Example:
  - If Honest(X) ⊃ (Sent(X,m) ⊃ Received(X,m')) and
     Y receives a message from X, then Y can conclude
     Honest(X) ⊃ Received(X,m')

### **Correctness of Challenge-Response**

 $\begin{array}{l} \mathsf{CR} \mid - [ \mathsf{InitCR}(\mathsf{A}, \mathsf{B}) ]_{\mathsf{A}} \mathsf{Honest}(\mathsf{B}) \ \supset \mathsf{ActionsInOrder}(\\ & \mathsf{Send}(\mathsf{A}, \{\mathsf{A},\mathsf{B},\mathsf{m}\}),\\ & \mathsf{Receive}(\mathsf{B}, \{\mathsf{A},\mathsf{B},\mathsf{m}\}),\\ & \mathsf{Send}(\mathsf{B}, \{\mathsf{B},\mathsf{A},\{\mathsf{n}, \mathsf{sig}_{\mathsf{B}}\ \{\mathsf{m}, \mathsf{n}, \mathsf{A}\}\}\}),\\ & \mathsf{Receive}(\mathsf{A}, \{\mathsf{B},\mathsf{A},\{\mathsf{n}, \mathsf{sig}_{\mathsf{B}}\ \{\mathsf{m}, \mathsf{n}, \mathsf{A}\}\}\}) \end{array}$ 

### 1: A Reasons about Own Actions

InitCR(A, X) = [
 new m;
 send A, X, {m, A};
 receive X, A, {x, sig<sub>x</sub>{m, x, A}};
 send A, X, sig<sub>A</sub>{m, x, X};

RespCR(B) = [
 receive Y, B, {y, Y};
 new n;
 send B, Y, {n, sig<sub>B</sub>{y, n, Y}};
 receive Y, B, sig<sub>Y</sub>{y, n, B};

### CR |- [InitCR(A, B)]<sub>A</sub> $\Diamond$ Verify(A, sig<sub>B</sub>{m, n, A})

If A completed a protocol session, it must have verified B's signature at some point

### 2: Properties of Signatures

InitCR(A, X) = [
 new m;
 send A, X, {m, A};
 receive X, A, {x, sig<sub>x</sub>{m, x, A}};
 send A, X, sig<sub>A</sub>{m, x, X};

RespCR(B) = [
 receive Y, B, {y, Y};
 new n;
 send B, Y, {n, sig<sub>B</sub>{y, n, Y}};
 receive Y, B, sig<sub>Y</sub>{y, n, B};

### CR |- [InitCR(A, B)]<sub>A</sub> Honest(B) $\supset$ $\exists$ t' ( $\diamond$ Send(B, t') $\land$ Contains(t', sig<sub>B</sub>{m, n, A})

If A completed protocol and B is honest, then B must have sent its signature as part of some message

### Honesty Invariant

InitCR(A, X) = [
 new m;
 send A, X, {m, A};
 receive X, A, {x, sig<sub>x</sub>{m, x, A}};
 send A, X, sig<sub>A</sub>{m, x, X};

RespCR(B) = [
 receive Y, B, {y, Y};
 new n;
 send B, Y, {n, sig<sub>B</sub>{y, n, Y}};
 receive Y, B, sig<sub>Y</sub>{y, n, B};

the protocol

This condition disambiguates  $sig_x(...)$  sent by responder from  $sig_A(...)$  sent by initiator

### Reminder: Honesty Rule

 $\forall$ roles R of Q.  $\forall$  initial segments A ⊆ R. Q |- [A]<sub>X</sub>  $\phi$ Q |- Honest(X) ⊃  $\phi$ 

### 3: Use Honesty Rule

InitCR(A, X) = [
 new m;
 send A, X, {m, A};
 receive X, A, {x, sig<sub>x</sub>{m, x, A}};
 send A, X, sig<sub>A</sub>{m, x, X};

RespCR(B) = [
 receive Y, B, {y, Y};
 new n;
 send B, Y, {n, sig<sub>B</sub>{y, n, Y}};
 receive Y, B, sig<sub>Y</sub>{y, n, B};

CR |- [InitCR(A, B)]<sub>A</sub> Honest(B)  $\supset$  $\Diamond$ Receive(B, {A,B,{m,A}})

> If A completed protocol and B is honest, then B must have received A's first message

## 4: Nonces Imply Temporal Order

]

InitCR(A, X) = [
 new m;
 send A, X, {m, A};
 receive X, A, {x, sig<sub>x</sub>{m, x, A}};
 send A, X, sig<sub>A</sub>{m, x, X};

]

RespCR(B) = [
 receive Y, B, {y, Y};
 new n;
 send B, Y, {n, sig<sub>B</sub>{y, n, Y}};
 receive Y, B, sig<sub>Y</sub>{y, n, B};

### CR |- [InitCR(A, B)]<sub>A</sub> Honest(B) $\supset$ ActionsInOrder(...)

### **Complete Proof**

$\mathbf{AM1}$	$(A \mathrel{B} \eta)[\ ]_{A,\eta} \operatorname{Has}(A,A,\eta) \wedge \operatorname{Has}(A,B,\eta)$
AN3	$[(\nu m)]_{A,\eta}$ Fresh $(A, m, \eta)$
AA1	$[\langle A, B, m \rangle]_{A,\eta} \diamondsuit \text{Send}(A, \{A, B, m\}, \eta)$
AA1	$[(B, A, n, \{m, n, A \}_{\overline{B}})]_{A,n}$
	$\bigotimes$ Receive $(A, \{B, A, n, \{ m, n, A \}_{\overline{B}}\}, \eta)$
AA1	$[(\{[m, n, A]\}_{\overline{B}}/\{[m, n, A]\}_{B})]_{A,\eta} \diamondsuit \text{Verify}(A, \{[m, n, A]\}_{\overline{B}}, \eta)$
AA1	$[\langle A, B, \{m, n, B\}_{\overline{A}} \rangle]_{A, \eta} \diamondsuit Send(A, \{A, B, \{m, n, B\}_{\overline{A}}\}, \eta)$
AF1, AF2	$(A B \eta)[(\nu m)\langle A, B, m \rangle(x)(x/B, A, n, \{m, n, A\}_{\overline{B}})$
	$(\{m, n, A\}_{\overline{B}}/\{[m, n, A]\}_{B})\langle A, B, \{m, n, B\}_{\overline{A}}\rangle]_{A, n}$
	ActionsInOrder(Send( $A, \{A, B, m\}, \eta$ ), Receive( $A, \{B, A, n, \{[m, n, A]\}_{\overline{B}}\}, \eta$ ),
	$Send(A,\{A,B,\{\!\mid\! m,n,B\}\!\}_{\overline{A}}\},\eta))$
$\mathbf{N1}$	$\diamondsuit$ New $(A, m, \eta) \supset \neg$ $\diamondsuit$ New $(B, m, \eta')$
5, VER	$Honest(B) \land \diamondsuit Verify(A, \{\![m, n, A]\!]_{\overline{B}}, \eta) \supset$
	$\exists \eta'. \exists m'. (\diamondsuit CSend(B,m',\eta') \land (\{\!\!\{m,n,A\}\!\!\}_{\overline{B}} \subseteq m'))$
HON	$Honest(B) \supset (\exists \eta'. \exists m'. ((\diamondsuit CSend(B, m', \eta') \land$
	$\{\! m,n,A \}_{\overline{B}} \subseteq m' \land \neg \diamondsuit New(B,m,\eta')) \supset$
	$(m' = \{B, A, \{n, \{m, n, A\}_{\overline{B}}\}\} \land \diamondsuit Receive(B, \{A, B, m\}, \eta') \land$
	$ActionsInOrder(Receive(B, \{A, B, m\}, \eta'), New(B, n, \eta'),$
	$Send(B, \{B, A, \{n, \{m, n, A \}_{\overline{B}}\}\}, \eta')))))$
2, 3, 11, AF3	$Honest(B) \supset After(Send(A, \{A, B, m\}, \eta),$
	$Receive(B, \{A, B, m\}, \eta'))$
$11, \mathbf{AF2}$	$Honest(B) \supset After(Receive(B, \{A, B, m\}, \eta'),$
	$Send(B, \{B, A, \{n, \{ m, n, A \}_{\overline{B}}\}\}, \eta'))$
11, 4, AF3	$Honest(B) \supset After(Send(B, \{B, A, \{n, \{m, n, A\}_{\overline{B}}\}\}, \eta'),$
	$Receive(A, \{B, A, \{n, \{[m, n, A]\}_{\overline{B}}\}\}, \eta))$
$10-13, \mathbf{AF2}$	$Honest(B) \supset \exists \eta'. (ActionsInOrder(Send(A, \{A, B, m\}, \eta),$
	$Receive(B, \{A, B, m\}, \eta'), Send(B, \{B, A, \{n, \{m, n, A\}\}_{\overline{B}}\}\}, \eta'),$
	$Receive(A, \{B, A, \{n, \{[m, n, A]\}_{\overline{B}}\}\}, \eta))$

#### Table 8. Deductions of A executing Init role of CR

### **Properties of Proof System**

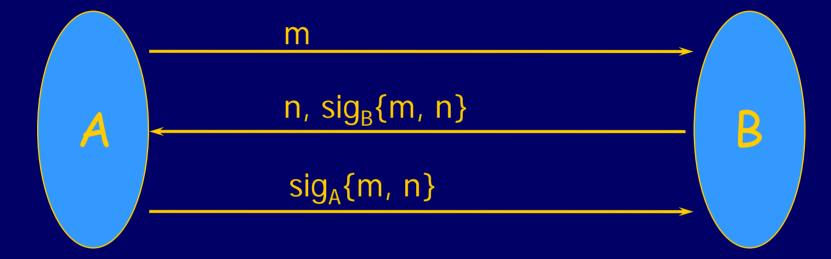
#### Soundness

- If φ is a theorem, then φ is a valid formula
   Q |- φ implies Q |= φ
- Informally: if we can prove something in the logic, then it is actually true

Proved formula holds in any step of any run

- There is no bound on the number of sessions!
- Unlike finite-state checking, the proved property is true for the entire protocol, not for specific session(s)

### Weak Challenge-Response



]

```
InitWCR(A, X) = [
```

]

```
new m;
send A, X, {m};
receive X, A, {x, sig<sub>x</sub>{m, x}};
send A, X, sig<sub>A</sub>{m, x};
```

RespWCR(B) = [
 receive Y, B, {y};
 new n;
 send B, Y, {n, sig<sub>B</sub>{y, n}};
 receive Y, B, sig<sub>Y</sub>{y, n};

### 1: A Reasons about Own Actions

]

```
InitWCR(A, X) = [
    new m;
    send A, X, {m};
    receive X, A, {x, sig<sub>x</sub>{m, x}};
    send A, X, sig<sub>A</sub>{m, x};
```

]

```
RespWCR(B) = [
    receive Y, B, {y};
    new n;
    send B, Y, {n, sig<sub>B</sub>{y, n}};
    receive Y, B, sig<sub>Y</sub>{y, n};
```

```
WCR |- [InitWCR(A, B)]<sub>A</sub>
\DiamondVerify(A, sig<sub>B</sub>{m, n})
```

### 2: Properties of Signatures

```
InitWCR(A, X) = [
    new m;
    send A, X, {m};
    receive X, A, {x, sig<sub>x</sub>{m, x}};
    send A, X, sig<sub>A</sub>{m, x}};
```

]

```
RespWCR(B) = [
    receive Y, B, {y};
    new n;
    send B, Y, {n, sig<sub>B</sub>{y, n}};
    receive Y, B, sig<sub>Y</sub>{y, n}};
```

WCR |- [InitWCR(A, B)]<sub>A</sub> Honest(B)  $\supset$  $\exists t' ( \diamondsuit Send(B, t') \land$ Contains(t', sig<sub>B</sub>{m, n})

]

### Honesty Invariant

```
InitWCR(A, X) = [
    new m;
    send A, X, {m};
    receive X, A, {x, sig<sub>x</sub>{m, x}};
    send A, X, sig<sub>A</sub>{m, x};
```

```
RespWCR(B) = [
    receive Y, B, {y};
    new n;
    send B, Y, {n, sig<sub>B</sub>{y, n}};
    receive Y, B, sig<sub>Y</sub>{y, n};
```

recipient Y

```
WCR |- Honest(X) \land
\diamondsuit Send(X, t') \land Contains(t', sig_{x}{y, x}) \land
\neg \diamondsuit New(X, y) \supset
\diamondsuit Receive(X, {Y, X, {y}}) In this protocol, sig_{x}{y,x}
does not explicitly include
identity of intended
```

### 3: Use Honesty Rule

InitWCR(A, X) = [
 new m;
 send A, X, {m};
 receive X, A, {x, sig<sub>x</sub>{m, x}};
 send A, X, sig<sub>A</sub>{m, x};

RespWCR(B) = [
 receive Y, B, {y};
 new n;
 send B, Y, {n, sig<sub>B</sub>{y, n}};
 receive Y, B, sig<sub>Y</sub>{y, n};

WCR |- [InitWCR(A, B)]<sub>A</sub> Honest(B)  $\supset$  $\Diamond$ Receive(B, {Y,B,sig<sub>Y</sub>{y,n}})

> B receives 3<sup>rd</sup> message from someone, not necessarily A

### Failed Proof and Counterexample

 WCR does not provide the strong authentication property for the initiator

- Counterexample: intruder can forge sender's and receiver's identity in first two messages
  - A -> X(B) A, B, m
  - X(C) -> B C, B, m [X pretends to be C]
  - B -> X(C) n, sig<sub>B</sub>(m, n)
  - X(B) -> A
     n, sig<sub>B</sub>(m, n)

### Further Work on Protocol Logic

- See papers by Datta, Derek, Mitchell, and Pavlovic on the course website
  - With a Diffie-Hellman primitive, prove authentication and secrecy for key exchange (STS, ISO-97898-3)
  - With symmetric encryption and hashing, prove authentication for ISO-9798-2, SKID3
- Work on protocol derivation
  - Build protocols by combining standard parts
    - Similar to the derivation of JFK described in class
  - Reuse proofs of correctness for building blocks
    - Compositionality pays off!