

Game-Based Verification of Contract Signing Protocols

Alternating Transition Systems

◆ Game variant of Kripke structures

- *R. Alur, T. Henzinger, O. Kupferman. "Alternating-time temporal logic". FOCS 1997.*

◆ Start by defining state space of the protocol

- Π is a set of propositions
- Σ is a set of players
- Q is a set of states
- $Q_0 \subseteq Q$ is a set of initial states
- $\pi: Q \rightarrow 2^\Pi$ maps each state to the set of propositions that are true in the state

◆ So far, this is very similar to $\text{Mur}\phi$

Transition Function

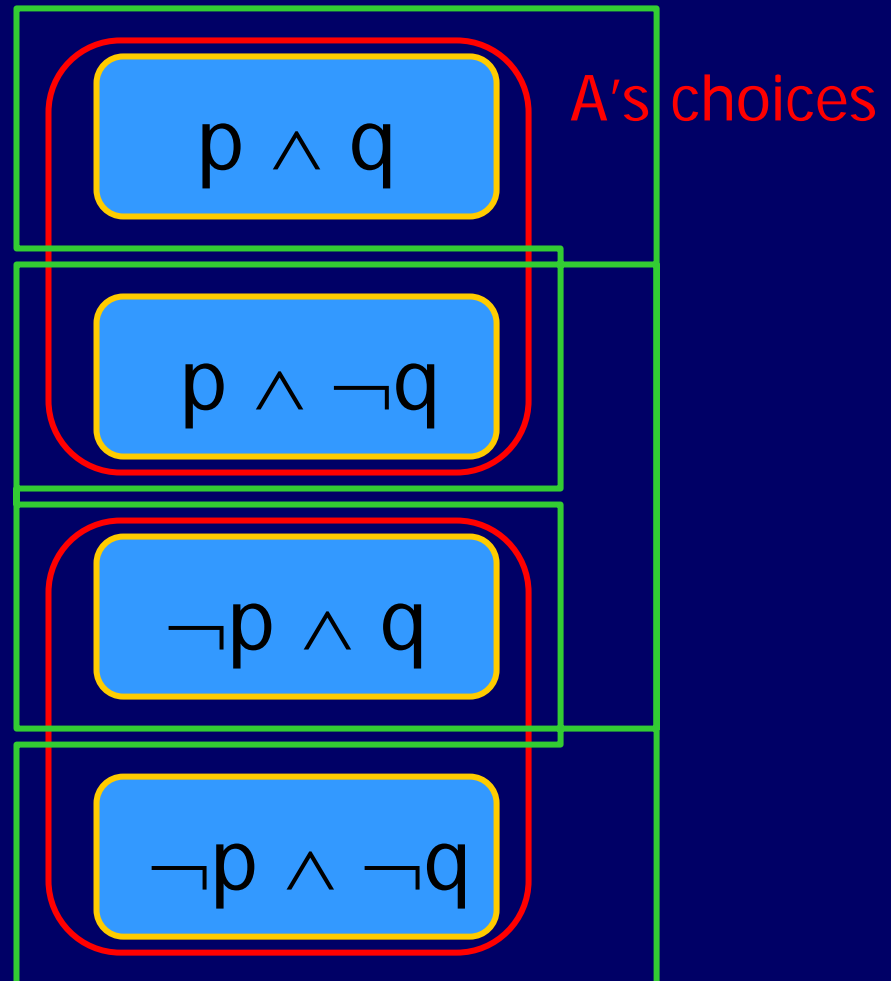
- ◆ $\delta: Q \times \Sigma \rightarrow 2^{2^Q}$ maps a state and a player to a nonempty set of choices, where each choice is a set of possible next states
 - When the system is in state q , each player chooses a set $Q_a \in \delta(q, a)$
 - The next state is the intersection of choices made by all players $\bigcap_{a \in \Sigma} \delta(q, a)$
 - The transition function must be defined in such a way that the intersection contains a unique state
- ◆ Informally, a player chooses a set of possible next states, then his opponents choose one of them

Example: Two-Player ATS

$\Sigma = \{\text{Alice}, \text{Bob}\}$

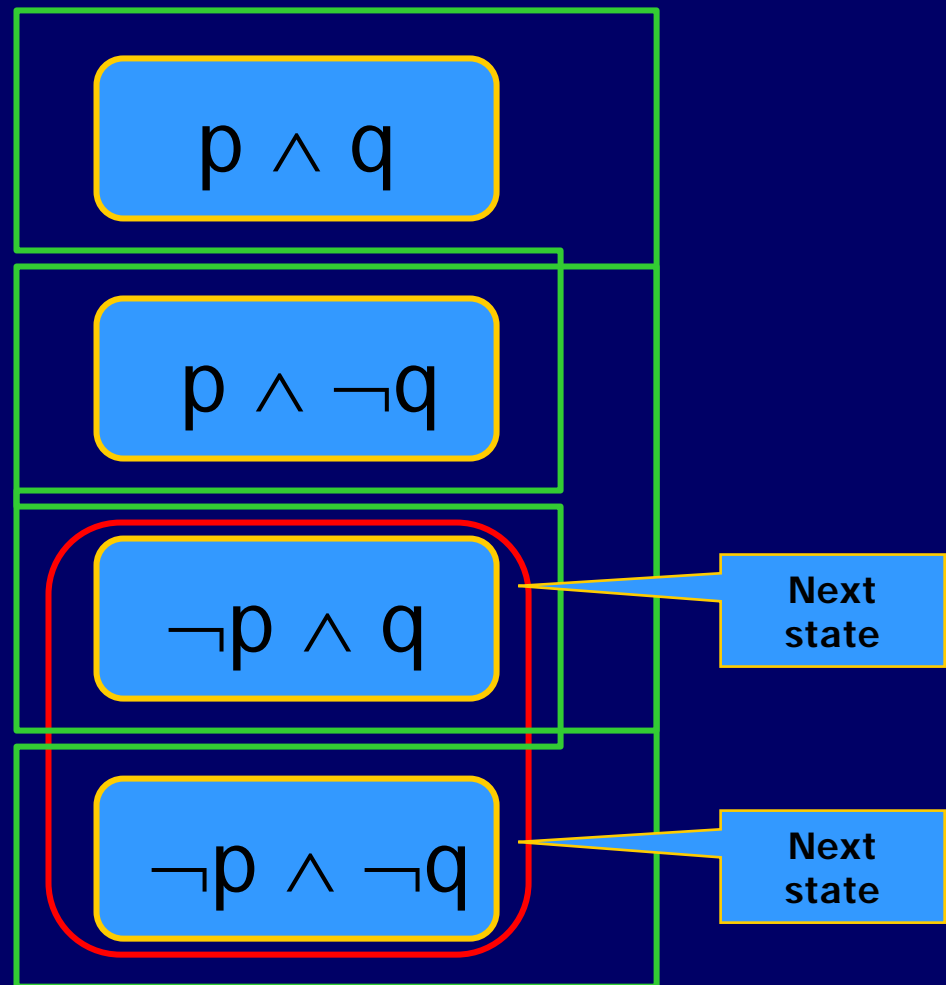
$p \wedge q$

B's choices



Example: Computing Next State

$\Sigma = \{\text{Alice}, \text{Bob}\}$



If A chooses this set...
... B can choose either state

Alternating-Time Temporal Logic

- ◆ Propositions $p \in \Pi$
- ◆ $\neg\varphi$ or $\varphi_1 \vee \varphi_2$ where $\varphi, \varphi_1, \varphi_2$ are ATL formulas
- ◆ $\langle\langle A \rangle\rangle \bigcirc \varphi$, $\langle\langle A \rangle\rangle \square \varphi$, $\langle\langle A \rangle\rangle \varphi_1 \cup \varphi_2$ where $A \subseteq \Sigma$ is a set of players, $\varphi, \varphi_1, \varphi_2$ are ATL formulas
 - These formulas express the ability of coalition A to achieve a certain outcome
 - \bigcirc , \square , \cup are standard temporal operators (similar to what we saw in PCTL)
- ◆ Define $\langle\langle A \rangle\rangle \diamond \varphi$ as $\langle\langle A \rangle\rangle \text{true} \cup \varphi$

Strategies in ATL

- ◆ A strategy for a player $a \in \Sigma$ is a mapping $f_a: Q^+ \rightarrow 2^Q$ such that for all prefixes $\lambda \in Q^*$ and all states $q \in Q$, $f_a(\lambda \cdot q) \in \delta(q, a)$
 - For each player, strategy maps any sequence of states to a set of possible next states
- ◆ Informally, the strategy tells the player in each state what to do next
 - Note that the player cannot choose the next state. He can only choose a set of possible next states, and opponents will choose one of them as the next state.

Temporal ATL Formulas (I)

- ◆ $\langle\langle A \rangle\rangle \bigcirc \varphi$ iff there exists a set F_a of strategies, one for each player in A , such that for all future executions $\lambda \in \text{out}(q, F_a)$ φ holds in first state $\lambda[1]$
 - Here $\text{out}(q, F_a)$ is the set of all future executions assuming the players follow the strategies prescribed by F_a , i.e., $\lambda = q_0 q_1 q_2 \dots \in \text{out}(q, F_a)$ if $q_0 = q$ and $\forall i \ q_{i+1} \in \bigcap_{a \in A} f_a(\lambda[0, i])$
- ◆ Informally, $\langle\langle A \rangle\rangle \bigcirc \varphi$ holds if coalition A has a strategy such that φ always holds in the next state

Temporal ATL Formulas (II)

- ◆ $\langle\langle A \rangle\rangle \Box \varphi$ iff there exists a set F_a of strategies, one for each player in A , such that for all future executions $\lambda \in \text{out}(q, F_a)$ φ holds in all states
 - Informally, $\langle\langle A \rangle\rangle \Box \varphi$ holds if coalition A has a strategy such that φ holds in every execution state
- ◆ $\langle\langle A \rangle\rangle \Diamond \varphi$ iff there exists a set F_a of strategies, one for each player in A , such that for all future executions $\lambda \in \text{out}(q, F_a)$ φ eventually holds in some state
 - Informally, $\langle\langle A \rangle\rangle \Diamond \varphi$ holds if coalition A has a strategy such that φ is true at some point in every execution

Protocol Description Language

◆ Guarded command language

- Very similar to PRISM input language (proposed by the same people)

◆ Each action described as `[] guard → command`

- `guard` is a boolean predicate over state variables
- `command` is an update predicate, same as in PRISM
- Simple example:

```
[]SigM1B ^ ¬SendM2 ^ ¬StopB -> SendMrB1' :=true;
```

Fairness in ATL

$$\neg \langle\langle B, Com \rangle\rangle \diamond (\text{contract}_A \wedge \neg \langle\langle A_h \rangle\rangle \diamond \text{contract}_B)$$


Bob in collaboration with communication channels
does not have a strategy
to reach a state in which
Bob has Alice's signature

but honest Alice does not have a strategy

to reach a state in which Alice has Bob's signature

Timeliness + Fairness in ATL

$\langle\langle A_h \rangle\rangle \diamond (\text{stop}_A \wedge (\neg \text{contract}_B \rightarrow \neg \langle\langle B, \text{Com} \rangle\rangle \diamond \text{contract}_A))$

Honest Alice always has a strategy to reach a state in which she can stop the protocol and if she does not have Bob's signature

then Bob does not have a strategy to obtain Alice's signature even if he controls communication channels

Abuse-Freeness in ATL

$$\neg \langle\langle A \rangle\rangle \diamond (\text{proveToC} \wedge \langle\langle A \rangle\rangle \diamond \text{contract}_B \wedge \langle\langle A \rangle\rangle \diamond (\text{aborted} \wedge \neg \langle\langle B_h \rangle\rangle \diamond \text{contract}_A))$$

Alice doesn't have a strategy to reach state in which she can prove to Charlie that she has a strategy to obtain Bob's signature AND a strategy to abort the protocol, i.e., reach a state where Alice has received abort token and Bob doesn't have a strategy to obtain Alice's signature

Modeling TTP and Communication

◆ Trusted third party is impartial

- This is modeled by defining a **unique TTP strategy**
- TTP has no choice: in every state, the next action is uniquely determined by its sole strategy

◆ Can model protocol under different assumptions about communication channels

- **Unreliable**: infinite delay possible, order not guaranteed
 - Add “idle” action to the channel state machine
- **Resilient**: finite delays, order not guaranteed
 - Add “idle” action + special constraints to ensure that every message is eventually delivered (rule out infinite delay)
- **Operational**: immediate transmission

MOCHA Model Checker

- ◆ Model checker specifically designed for verifying alternating transition systems
 - System behavior specified as guarded commands
 - Essentially the same as PRISM input, except that transitions are nondeterministic (as in in $\text{Mur}\phi$), not probabilistic
 - Property specified as ATL formula
- ◆ Slang scripting language
 - Makes writing protocol specifications easier
- ◆ Try online implementation!
 - <http://www-cad.eecs.berkeley.edu/~mocha/trial/>