

Quiz, CS 336
November 24, 2009

Show all your work for these problems.

1. Consider the following pseudo-code program for computing $X = \sum_{i=1}^n a[i]$, where $a[1..n]$ is an input array of integers, and $n \geq 1$.

```
X := 0
i := 1
While (i ≤ n) DO
    X := X + a[i]
    i := i + 1
End(While)
Return(X)
```

Answer the following questions:

- Prove that $i \leq n + 1$ is a loop invariant
 - Prove that $X = a[1] + a[2] + \dots + a[i - 1]$ is a loop invariant.
 - Show that the returned value of X is as desired ($a[1] + a[2] + \dots + a[n]$), under the assumption that the conditions above are loop invariants.
2. Consider strings from the alphabet $\{A, C, T, G\}$ (DNA alphabet).
 - For a given $k \geq 1$, how many strings of length k are there?
 - How many strings of length 3 are there where they are all the same letter?
 - How many strings of length 3 are there where at least two letters are the same?
 - How many strings of length 3 are there that satisfy at least one of the following constraints:
 - Begin with A
 - Have C in the middle position
 - Have T in the third position