Quiz #2, CS 336, Fall 2009

Instructions: Show all your work.

1. Write down the algorithm for computing the length of the longest increasing subsequence given input $a_1, a_2, \ldots, a_t$, where each a_i is an integer and $t \geq 1$. Answer the following questions:

   (a) What do your variables mean? (In English, please.)

   (b) Once all your variables are computed, what do you return as your output?

   (c) How do you initialize the algorithm? (What is your base case?)

   (d) What is the order in which you will calculate the values for your variables?

   (e) Apply the algorithm for the first problem to the following input

   $$5, 1, 13, 8, 2, 65, 3, 10, -5, 7, 21, 4, 3, 0$$

   and show the values for all the variables.

   (f) Give one longest increasing subsequence for that input.

   (g) What is the running time of your algorithm, and why? (Give a justification for your analysis.)

2. Consider the following recursively defined function:

   - $T(1) = 1$, and
   - $T(n) = T(n-1) + 2n + 1$ if $n > 1$.

   Prove, by induction, that $T(n) \leq 3n^2$ for all integers $n \geq 1$.