Quiz 5, CS 336, Tandy Warnow

Give your solution for each problem. No explanation needed.

1. Consider a set X of N people, where X includes Henry, Sally, and Bob (so $N \geq 3$). Express your answers as a function of N.

   (a) How many subsets of X are there?

   (b) How many subsets of X have exactly three people?

   (c) How many subsets of X have at least three people?

   (d) How many subsets of X have both Henry and Sally?

   (e) How many subsets of X have neither Henry nor Sally?

   (f) How many subsets of X have at most one of Henry or Sally?

   (g) How many ways can you select three people from X, and put them in a row?

2. Prove by induction on n that $F(n) \geq n$ for all $n \geq 1$ where $F(n)$ is defined by $F(1) = 1, F(2) = 2$ and $F(n) = F(n-2) + 3$ when $n \geq 3$.

3. Consider a sequence of sets $A_1, A_2, A_3, \ldots$ where $A_1 = \{1\}$ and $A_n = A_{n-1} \cup \{2n, 2n-1\}$ where $n \geq 2$.

   (a) What is A_2?

   (b) What is A_3?

   (c) Prove, by induction on n, that $|A_n| = 2n - 1$.