Achievements in Answer Set Programming

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Abstract
This paper describes an approach to the methodology of answer set programming that can facilitate the design of encodings that are easy to understand and provably correct. Under this approach, after appending a rule or a small group of rules to the emerging program we include a comment that states what has been “achieved” so far. This strategy allows us to set out our understanding of the design of the program by describing the roles of small parts of the program in a mathematically precise way.

1 Introduction
This paper describes an approach to the methodology of answer set programming (Marek and Truszczyński 1999; Niemelä 1999) that can facilitate the design of encodings that are easy to understand and provably correct. Under this approach, after appending a rule or a small group of rules to the emerging program, the programmer would include a comment that states what has been “achieved” so far, in a certain precise sense.

Consider, for instance, the following solution to the 8 queens problem, adapted from (Gebser et al. 2012, Section 3.2).

```
1 % Program 8Queens
2 3 row(1..8).
4 col(1..8).
5 8 { queen(I,J) : col(I), row(J) } 8.
6 :- queen(I,J), queen(I,JJ), J!=JJ.
7 :- queen(I,J), queen(II,J), I!=II.
8 :- queen(I,J), queen(II,JJ), (I,J)!=(II,JJ), |I-II|=|J-JJ|.
```

The first rule of \textit{NQueens} (Line 3), viewed as a one-rule program, has a unique stable model \(S\), which satisfies the following condition:

A ground atom of the form \textit{row}(i) belongs to \(S\) iff \(i \in \{1,\ldots,8\}\). \hfill (1)

Condition (1) holds also if \(S\) is the stable model of the first two rules of this program. And it holds if \(S\) is any stable model of the first three rules, and so on, for all 6 “prefixes” (initial segments) of the program. This is what we mean by achievement: once the programmer declares that a

\footnote{Examples of ASP programs in the paper are written in the input language of the grounder \textsc{Gringo}, Version 5.}
property “has been achieved,” he is committed to maintaining this property of stable models until the program is completed.

After writing the second rule (Line 4), the programmer can claim that something else has been achieved:

A ground atom of the form \( \text{col}(j) \) belongs to \( S \) iff \( j \in \{1, \ldots, 8\} \). \hspace{1cm} (2)

This condition holds if \( S \) is a stable model of any prefix of the program that includes the first two rules.

Additional properties achieved by adding the third rule can be expressed as follows:

Set \( S \) contains exactly 8 ground atoms of the form \( \text{queen}(i, j) \).

For each of these atoms, \( i, j \in \{1, \ldots, 8\} \). \hspace{1cm} (3)

If a program is written in this manner then every achievement documented in the process of writing it describes a property shared by all stable models of the entire program. In some cases this list of achievements can serve as the skeleton of a proof of its correctness, in the spirit of Edsger Dijkstra’s advice:

… one should not first make the program and then prove its correctness, because then the requirement of providing the proof would only increase the poor programmer’s burden. On the contrary: the programmer should let correctness proof and program grow hand in hand (Dijkstra 1972).

Recording important achievements in the process of writing an ASP program may be similar to recording important loop invariants in procedural programming: it does not ensure the correctness of the program but helps the programmer move toward the goal of proving correctness.

A preliminary report on this project was presented at the 2016 Workshop on Answer Set Programming and Other Computing Paradigms.

2 Programs, Prefixes, and Achievements

In this paper, by an (ordered) program we understand a list of rules \( R_1, \ldots, R_n \) (\( n \geq 1 \)) in the input language of an answer set solver, such as CLINGO (Gebser et al. 2015) or DLV (Eiter et al. 1998). The order of rules is supposed to reflect the order in which the programmer writes them in the process of creating the program. It does not affect the semantics of the program, but it is essential for understanding the process of programming.

We restrict attention to programs without classical negation. (This limitation is discussed in the conclusion.) Stable models of a program without classical negation are sets of ground atoms that contain no arithmetic operations, intervals, or pools (Gebser et al. 2015, Sections 3.1.7, 3.1.9, 3.1.10). Such ground atoms will be called precomputed.\(^2\) An interpretation is a set of precomputed atoms.

The k-th prefix of a program \( R_1, \ldots, R_n \), where \( 1 \leq k \leq n \), is the program \( R_1, \ldots, R_k \). We will express that a program \( \Gamma \) is a prefix of a program \( \Pi \) by writing \( \Gamma \leq \Pi \). The relation \( \leq \) is a total order on the set of prefixes of a program.

In some programs, constants are used as placeholders for values provided by the user (Gebser et al. 2015, Section 3.1.15). For example, the constant \( n \) is used as a placeholder for an arbitrary positive integer in the following more general version of 8Queens:

\(^2\) This terminology follows Gebser et al. (2012, Section 2.1), where “precomputed terms” are defined. Calimeri et al. (2012, Section 2.1) talk about elements of the “Herbrand universe” of a program in the same sense.
% Program NQueens
row(1..n).
col(1..n).
n { queen(I,J) : col(I), row(J) } n.
:- queen(I,J), queen(I,JJ), J!=JJ.
:- queen(I,J), queen(II,J), I!=II.
:- queen(I,J), queen(II,JJ), (I,J)!=(II,JJ), |I-II|=|J-JJ|.

We assume here that \( \Pi \) is a program that does not contain such placeholders, so that the stable models of \( \Pi \) are completely determined by its rules, without any additional input from the user. An extension to programs with input is discussed in Section 3.

An achievement of a prefix \( \Gamma \) of \( \Pi \) is a property of sets of interpretations that holds for all stable models of all programs \( \Delta \) such that \( \Gamma \leq \Delta \leq \Pi \). For example, (1) is an achievement of the first prefix of program 8Queens; (2) is an achievement of its second prefix; and (3) is an achievement of its third prefix. Conditions (1) and (2), and the conjunction of conditions (1)–(3), are achievements of the third prefix as well. Any condition that holds for all sets of interpretations is trivially an achievement of any prefix of any program.

The following three conditions are achievements of the last three prefixes of 8Queens:

Each column of the \( 8 \times 8 \) chessboard includes at most one square \( (i,j) \) such that the atom \( \text{queen}(i,j) \) belongs to \( S \).

Each row of the \( 8 \times 8 \) chessboard includes at most one square \( (i,j) \) such that the atom \( \text{queen}(i,j) \) belongs to \( S \).

Each diagonal of the \( 8 \times 8 \) chessboard includes at most one square \( (i,j) \) such that the atom \( \text{queen}(i,j) \) belongs to \( S \).

Thus every stable model \( S \) of 8Queens satisfies all conditions (1)–(6).

3 Programs with Input

The value of a placeholder, such as constant \( n \) in the NQueens program above, is one kind of input that an answer set solver may expect in addition to the rules of the program. A definition of an “extensional predicate” occurring in the bodies of rules is another kind. Consider, for example, the following encoding of Hamiltonian cycles, adapted from (Gebser et al. 2012, Section 3.3):

% Program Hamiltonian
1 {in(X,Y) : edge(X,Y) } 1 :- vertex(X).
1 {in(X,Y) : edge(X,Y) } 1 :- vertex(Y).
reached(X) :- in(v0,X).
reached(Y) :- reached(X), in(X,Y).
:- not reached(X), vertex(X).
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- the predicate $\text{vertex}/1$, representing the set of vertices of a finite digraph $G$,
- the predicate $\text{edge}/2$, representing the set of edges of $G$,
- the placeholder $v_0$, which is a vertex of $G$.

If $i$ is a valid input for a program $\Pi$ then we can talk about models of $\Pi$ that are stable for input $i$, or “$i$-stable.”3

For a program with input, an achievement is defined as a relation between valid inputs and sets of interpretations. Such a relation $R$ will be called an achievement of a prefix $\Gamma$ of $\Pi$ if $R(i,S)$ holds whenever $S$ is an $i$-stable model of a program $\Delta$ such that $\Gamma \leq \Delta \leq \Pi$.

For example, the sentence

\[
\text{A ground atom of the form } \text{row}(i) \text{ belongs to } S \text{ iff } i \in \{1,\ldots,n\}
\]

expresses a relation between $n$ and $S$ that is an achievement of the first prefix of $\text{NQueens}$. It is obtained from condition (1) by replacing 8 with $n$, and achievements of the other prefixes of that program can be obtained in a similar way from conditions (2)–(6).

The following two conditions are achievements of the first two prefixes of $\text{Hamiltonian}$:

Every pair $(x, y)$ such that the atom $\text{in}(x,y)$ belongs to $S$ is an edge of $G$; for every vertex $x$ of $G$ there is a unique $y$ such that the atom $\text{in}(x,y)$ belongs to $S$.

For every vertex $y$ of $G$ there is a unique $x$ such that the atom $\text{in}(x,y)$ belongs to $S$.

(8)

(9)

Nothing interesting has been achieved by adding the third rule, but the following condition is an achievement of the fourth prefix of the program:

The set of symbols $x$ such that the atom $\text{reached}(x)$ belongs to $S$ consists of the vertices $x$ for which there exists a walk $v_0, \ldots, v_n$ such that $n \geq 1$, $v_0 = v_0$, $v_n = x$, and every atom of the form $\text{in}(v_i,v_{i+1})$ belongs to $S$.

Finally, here is an achievement of the entire program:

For every vertex $x$ of $G$, the atom $\text{reached}(x)$ belongs to $S$.

(11)

4 Records of Achievement

A record of achievement for a program $\Pi$ is a function that maps some (possibly all) prefixes of $\Pi$, including $\Pi$ itself, to their achievements. For instance, the function that maps the prefixes of $\text{NQueens}$ to conditions (1)–(6) is a record of achievement, as well as the function that maps the first, second, fourth and fifth prefixes of $\text{Hamiltonian}$ to conditions (8)–(11). Every program has a trivial record of achievement that maps all its prefixes to the identically true condition.

A record of achievement $a$ can be represented by including in the program, after each prefix $\Gamma$ in the domain of $a$, a comment that describes the condition $a(\Gamma)$. Here is the program $\text{NQueens}$ with a record of achievement encoded in this way by comments, and with an additional comment at the beginning that shows which inputs for the program are considered valid:

3 Programs with input are similar to lp-functions in the sense of Gelfond and Przymusinska (1996).
% Program NQueens, with a record of achievement

% input: size n of the board

% A square on the board is represented as a pair,
% column number and row number, both from the set
% \{1,\ldots, n\}.

row(1..n).

% achieved: row/1 = \{1,\ldots, n\}.

col(1..n).

% achieved: col/1 = \{1,\ldots, n\}.

\( n \{ \text{queen}(I,J) : \text{col}(I), \text{row}(J) \} \in n. \)

% achieved: Set queen/2 consists of n squares.

\( :- \text{queen}(I,J), \text{queen}(I,JJ), J \neq JJ. \)

% achieved: Each column includes at most one square from
% queen/2.

\( :- \text{queen}(I,J), \text{queen}(II,J), I \neq II. \)

% achieved: Each row includes at most one square from
% queen/2.

\( :- \text{queen}(I,J), \text{queen}(II,JJ), (I,J) \neq (II,JJ), |I-II| = |J-JJ|. \)

% achieved: Each diagonal includes at most one square from
% queen/2.

The comment in Line 10 is a concise reformulation of condition (7). It uses row/1 as shorthand for “the set of precomputed terms \( i \) such that the atom row(\( i \)) belongs to \( S \).” In the other comments, col/1 and queen/2 are understood in a similar way.

Program Hamiltonian with a record of achievement below uses another useful convention: in Lines 11 and 12, we understand \( X \) and \( Y \) as metavariables for precomputed terms. The comment in those lines is a reformulation of condition (9).

5 Completeness

If \( a \) is a record of achievement for a program \( \Pi \), and \( \Delta \) is a prefix of \( \Pi \), then by \( a^{\ast}(\Delta) \) we denote the conjunction of conditions \( a(\Gamma) \) for all prefixes \( \Gamma \) of \( \Delta \) that belong to the domain of \( a \). It is clear that condition \( a^{\ast}(\Delta) \) is an achievement of \( \Delta \). In particular, all stable models of \( \Pi \) satisfy condition \( a^{\ast}(\Pi) \). The converse is, generally, not true—it is possible that an interpretation satisfying \( a^{\ast}(\Pi) \) is not a stable model of \( \Pi \).
% Program Hamiltonian, with a record of achievement

% input: the set vertex/1 of vertices of a finite digraph G;
% the set edge/2 of edges of G; a vertex v0 of G.

1 \{\text{in}(X,Y) : \text{edge}(X,Y) \} 1 :- \text{vertex}(X).
% achieved: Set in/2 is a subset of edge/2; for every vertex
% X of G there is a unique Y such that in(X,Y).

1 \{\text{in}(X,Y) : \text{edge}(X,Y) \} 1 :- \text{vertex}(Y).
% achieved: For every vertex Y of G there is a unique X
% such that in(X,Y).

\text{reached}(X) :- \text{in(v0,X)}.
% achieved: Set reached/1 consists of the vertices that are
% reachable from v0 by a path of non-zero length
% in the subgraph of G with the set of edges in/2.

\text{reached}(Y) :- \text{reached}(X), \text{in}(X,Y).

\text{reached}(X) :- \text{in}(v0,X).
\text{reached}(Y) :- \text{reached}(X), \text{in}(X,Y).
% achieved: Set reached/1 consists of the vertices that are
% reachable from v0 by a path of non-zero length
% in the subgraph of G with the set of edges in/2.

:- \text{not reached}(X), \text{vertex}(X).
% achieved: reached/1 = vertex/1.

The following notation and terminology will be used to discuss this issue. For any prefix \( \Gamma \) of
a program \( \Pi \), by \( \text{Preds}(\Gamma) \) we denote the set consisting of the predicates occurring in \( \Gamma \) and the
input predicates of \( \Pi \). For instance, if \( \Gamma \) is the first prefix of \text{Hamiltonian} (rule in Line 6) then
\( \text{Preds}(\Pi) \) consists of the predicates

\text{vertex}/1, \text{edge}/2, \text{in}/2. \hspace{2cm} (12)

The expression \( \text{Preds}(I) \), where \( I \) is an interpretation, stands for the set of predicates occurring
in \( I \). About a record of achievement \( a \) for a program \( \Pi \) we will say that it is complete if for
every \( \Gamma \) in the domain of \( a \), all interpretations \( I \) satisfying \( a^*(\Gamma) \) for which \( \text{Preds}(I) \subseteq \text{Preds}(\Gamma) \)
are stable models of \( \Gamma \).

The records of achievement for \text{NQueens} and \text{Hamiltonian} shown above are complete. The
completeness of the latter, for example, entails that

(a) any interpretation \( S \) such that atoms in \( S \) contain no predicates other than (12) is a stable
model of the first rule of \text{Hamiltonian} iff it satisfies condition (8);
(b) such an interpretation is a stable model of the first two rules of \text{Hamiltonian} iff it satisfies
conditions (8) and (9);
(c) any interpretation \( S \) such that atoms in \( S \) contain no predicates other than

\text{vertex}/1, \text{edge}/2, \text{in}/2, \text{reached}/1

is a stable model of the first four rules of \text{Hamiltonian} iff it satisfies conditions (8)–(10);
(d) such an interpretation is a stable model of the entire program iff it satisfies conditions
(8)–(11).
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6 Achievement-Based Answer Set Programming

Records of achievement in the two examples above are not only complete but also detailed, in the sense that they include achievements for almost all prefixes of the programs. The only rule in these programs that is not followed by an achievement comment is the first rule in the recursive definition of reached. The role of that rule cannot be properly explained unless we treat it as part of the definition.

Developing an ASP program along with a complete and detailed record of achievement can be called “achievement-based” answer set programming. This strategy allows us to set out our understanding of the design of the program by describing the roles of individual rules, or small groups of rules, in a mathematically precise way.

One of the advantages of this approach is that comments explaining what is achieved by a group of rules at the beginning of a program help us start testing and debugging it at an early stage, when only a part of the program has been written. To this end, we can run an answer set solver to find some (or all) stable models of the prefix that has been already written and check that they satisfy the conditions in the available “achieved” comments. A mismatch would indicate that there is a bug in the rules of the program written so far, or perhaps that the programmer’s intentions have not been properly documented.

A complete record of achievement is particularly valuable when it is closely related to the program’s specification, because from the completeness of such a record we may be able to conclude that the program is correct. For instance, from property (d) of Hamiltonian we can conclude that a set of edges of G is a Hamiltonian cycle iff it has the form \{ (x,y) : \text{in}(x,y) \in S \} for some stable model S of that program.

To further illustrate the idea of achievement-based ASP, we present below three “real life” ASP programs accompanied by complete, detailed records of achievement. The first of them, program SCA (Brain et al. 2012, Figure 1), generates sequence covering arrays of strength 3. Our version of SCA is slightly different from the original program: the constraint in Line 19 here replaces the pair of rules

\[
\begin{align*}
\text{hb(}N,X,Z) & : - \text{hb(}N,X,Y) , \text{ hb(}N,Y,Z) . \\
& : - \text{hb(}N,X,X) .
\end{align*}
\]

The reason why we chose to make this change is that the first of the two rules above may temporarily destroy the irreflexivity of the relation of \text{hb}_N that was true at the previous step; that property is restored by the second rule. That is not in the spirit of the achievement-based approach, which emphasizes the gradual accumulation of properties that we would like to see in the complete program.

The other two examples are program Borda, adapted from (Charwat and Pfandler 2015, Encoding 1), which encodes the Borda rule for determining the winner in an election with several candidates; and program OBT, adapted from (Brooks et al. 2007, Section 1), which encodes ordered binary trees.

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4 A sequence covering array of strength t is an array of permutations of symbols such that every ordering of any t symbols appears as a subsequence of at least one row.

5 Each voter ranks the list of candidates in order of preference. The candidate ranked last gets zero points; next to last gets one point, and so on. The candidate with the most points is the winner.

6 An ordered binary tree is a rooted binary tree with the leaves 0,...,k and internal vertices k+1,...,2k such that (i) every internal vertex is greater than its children, and (ii) for any two internal vertices x and x₁, x > x₁ iff the maximum of the children of x is greater than the maximum of the children of x₁.
% Program SCA

% input: the number s of symbols 1, ..., s; the number n of
% rows 1, ..., n.

sym(1..s).
% achieved: sym/1 = {1, ..., s}.

row(1..n).
% achieved: row/1 = {1, ..., n}.

\{hb(N,X,Y); hb(N,Y,X)\} 1 :- row(N), sym(X), sym(Y), X!=Y.
% For every row N, let hb_N be the binary relation on sym/1
% defined by the condition: X hb_N Y iff hb(N,X,Y).
% achieved: each relation hb_N is irreflexive; each pair of
% distinct symbols satisfies either X hb_N Y
% or Y hb_N X.

:- hb(N,X,Y), hb(N,Y,Z), not hb(N,X,Z).
% achieved: each relation hb_N is transitive.

covered(X,Y,Z) :- hb(N,X,Y), hb(N,Y,Z).
% For every row N and every symbol X, by M_{N,X} we denote
% the symbol that is the X-th smallest w.r.t. hb_N.
% achieved: for any symbols X, Y, Z, covered(X,Y,Z) iff,
% for some row N, (X,Y,Z) is a subsequence of
% (M_{N,1},...,M_{N,s}).

:- not covered(X,Y,Z), sym(X), sym(Y), sym(Z),
  X!=Y, Y!=Z, X!=Z.
% achieved: covered(X,Y,Z) for any pairwise distinct symbols
% X, Y, Z.
% Program Borda

% input: the number m of candidates 1,...,m in an election E; the set \( p/3 \) of triples \((P,\text{Pos},C)\) such that, for a fixed ordering \( pr_1,\ldots,pr_l \) of the distinct preference relations in the profile of E, candidate \( C \) is at position \( \text{Pos} \) in relation \( pr_P \); the set \( \text{votecount}/2 \) of pairs \((P,VC)\) such that relation \( pr_P \) occurs \( VC \) times in the profile of E.

candidate(1..m).
% achieved: candidate/1 = \{1,...,m\}.

posScore(P,C,X*VC) :- p(P,Pos,C), X=m-Pos, votecount(P,VC).
% achieved: posScore(P,C,S) iff the voters who chose relation \( pr_P \) in election E contributed \( S \) points to candidate \( C \) under the Borda rule.

score(C,N) :- candidate(C), N=# sum \{S: posScore(_,C,S)\}.
% achieved: score(C,N) iff candidate \( C \) earned \( N \) points in election E under the Borda rule.

winner(C) :- score(C,M), M=# max\{S: score(_,S)\}.
% achieved: winner(C) iff the number of points earned by candidate \( C \) in election E is maximal among all candidates.
% Program OBT

% input: positive integer k.

leaf(0..k).
% achieved: leaf/1 = {0,...,k}.

vertex(0..2*k).
% achieved: vertex/1 = {0,...,2k}.

internal(X) :- vertex(X), not leaf(X).
% achieved: internal/1 = {k+1,...,2k}.

2 \{ edge(X,Y) : vertex(Y), X>Y \} 2 :- internal(X).
% Let G be the digraph with the vertices vertex/1 and the
% edges edge/2.
% achieved: for every edge (X,Y) of G, X>Y; the out-degree
% of a vertex X in G is 2 if internal(X), and 0
% if leaf(X).

reachable(X,Y) :- edge(X,Y).
reachable(X,Y) :- edge(X,Z), reachable(Z,Y).
% achieved: reachable(X,Y) iff Y is reachable from X in G
% by a path of non-zero length.

:- vertex(X), X!=2*k, not reachable(2*k,X).
% achieved: every vertex of G other than 2k is reachable
% from 2k by a path of non-zero length.

:- reachable(X,X), vertex(X).
% achieved: G is acyclic.

max_child(X,Y) :- edge(X,Y), edge(X,Y1), Y > Y1.
% achieved: max_child(X,Y) iff Y is the largest child of X
% in G.

Y<Y1 :- max_child(X,Y), max_child(X1,Y1), Y>Y1, X<X1.
% achieved: for any vertices X, X1 of G such that X<X1, the
% largest child of X is smaller than the largest
% child of X1.
7 Achievements in Teaching

The achievement-based approach is emphasized in a class on answer set programming that is being taught to a group of over 50 undergraduates at the University of Texas at Austin, and it appears that many students find it useful. From the instructor’s perspective, “achieved” comments are useful for describing exercises and test problems. In one case, students were shown an “incomplete listing” of a graph coloring program:

```
% Color the vertices of a graph so that no two adjacent
% vertices share the same color.

% input: set vertex/1 of vertices of a graph G;
% set edge/2 of edges of G; set color/1 of colors.

1 { color(X,C) : color(C) } 1 :- vertex(X).
% achieved: for every vertex X there is a unique color C
% such that color(X,C).

% achieved: no two adjacent vertices share the same color.

#show color/2.
```

The question was, “What rule would you place in Line 11?” On another occasion, students were asked to write a one-rule program for which the following comments would be appropriate:

```
% Calculate the number of classes taught today on each of
% the seven floors of the computer science building.

% input: set where/2 of all pairs (C,I) such that class C
% is taught on the I-th floor.

% achieved: howmany(I,N) iff the number of classes taught
% on the I-th floor is N.

#show howmany/2.
```

8 Conclusion

In achievement-based ASP, we start writing a program by describing its inputs. Then, after every rule or small group of rules, we include a comment describing what has been achieved. Collectively these comments represent a complete, detailed record of achievement.

As we are adding rules to an emerging ASP program, we deal at every step with a single executable piece of code, unlike the non-executable pseudo-code formed in the process of stepwise refinement of a procedural program, and unlike a collection of executable subroutines formed in
the process of bottom-up design. In the process, we think of prefixes of the emerging program as if they were complete programs. We describe their stable models in a way that relates them to the stable models of the final product.

The programs discussed in this paper do not use classical negation (Gelfond and Lifschitz 1990). In the presence of classical negation, answer sets consist of “precomputed classical literals”—precomputed atoms and classical negations of such atoms. Extending the definition of a complete record of achievement to such programs is straightforward. On the other hand, many programs with classical negation contain defaults (Gelfond and Kahl 2014, Chap. 5), such as the closed world assumption and the commonsense law of inertia, and the achievement-based approach may be not so useful in application to programs containing defaults. A default does not “achieve” anything in the technical sense of Section 2.

According to Gebser et al. (2012, Section 3.2),

[the basic approach to writing encodings in ASP follows a generate-and-test methodology, also referred to as guess-and-check... A generating part is meant to non-deterministically provide solution candidates, while a testing part eliminates candidates violating some requirements... Both parts are usually amended by defining parts providing auxiliary concepts.

Most programs discussed in this paper are designed in accordance with this basic approach. The advice to keep track of what has been achieved as you are adding rules to your program differs from the “generate-and-test” advice in that it refers to mathematical properties of stable models, and not to programmer’s intentions.

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Program Borda is an exception—it has no generating part and no testing part. Also, it is not clear whether the designers of Hamiltonian intended the second rule for the generating part or for the testing part. (The second rule is syntactically similar to the first, which is definitely a generate rule. On the other hand, adding the second rule does not really generate new solution candidates; it eliminates some of the candidates generated earlier.)
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