ECWA Made Easy

Vladimir Lifschitz
Department of Computer Sciences
and Department of Philosophy
University of Texas at Austin
Austin, TX 78712

Abstract

We generalize a theorem on the relation of the Extended Closed World Assumption (ECWA) to circumscription due to Gelfond, Przymusinska and Przymusinski, and give a simple proof of the generalized result.

1 Introduction

This note adds another link to the chain of results that begins with the main theorem of Jack Minker's paper "On Indefinite Databases and the Closed World Assumption" [Minker, 1982].

Minker's work was motivated by the need to understand the relationship between two concepts of a database. One concept is "extensional": A database consists of data as stored in traditional relational databases. The other is "intensional," or "deductive": A database is a set of formulas (for instance, clauses), similar to the axiom set of a formal theory. From the deductive viewpoint, an extensional database is a collection of formulas of the simplest possible kind—ground atoms. The intensional approach allows us to represent, for instance, the fact that some binary relation is reflexive by one short axiom. In an extensional database, on the other hand, the same information would have to be represented by a large collection of data. In the late seventies, several researchers recognized the importance of the additional expressivity given by the deductive view, and the theory of deductive databases came into existence [Minker, 1988].

The claim that an extensional database is merely a set of atomic axioms ignores, however, an important detail. A set of ground atoms provides only positive information: Each atom tells us that some relation R holds for some tuple of objects $\overline{a}$. Any set of atomic axioms is, in fact, consistent with the assumption that all predicates are identically true. A representation of a predicate in an extensional database provides both positive and negative information: If a tuple $\overline{a}$ is not included among those that satisfy R, then we know that R does not hold for this tuple. This distinction was described by Ray Reiter as the difference between "open world" and "closed world" databases [Reiter, 1978]. Axiomatic theories of classical logic are in the first group, and traditional relational databases are in the second.

Reiter was interested primarily in the databases that are "definite," that is, contain no disjunctive information. Minker's goal was to generalize the closed world assumption to the case of indefinite databases.

In [Minker, 1982], we find two definitions of the generalized closed world assumption—"syntactic" and "semantic." Their equivalence is the main result of the paper. The semantic definition is based on the idea of a "minimal model"—the concept that, two years earlier, led John McCarthy to the theory of circumscription [McCarthy, 1980].

The relationship between the uses of minimal models by McCarthy and by Minker is far from being simple. Minker's definitions are, in a number of ways, less general. He restricts his attention to universal theories and their Herbrand models; he makes use of only one kind of minimality (there is no counterpart of "circumscription policies" in his paper); he is interested in deriving negative ground literals, rather than arbitrary sentences. In McCarthy's work, on the other hand, we find nothing similar to the syntactic formulation of the generalized closed world assumption.

This picture has been clarified by a number of researchers over the last ten years. Besides its theoretical interest, their work is important as a basis for the design of theorem provers for circumscription. One significant step was made by Michael Gelfond, Halina Przymusinska and Teodor Przymusinski, who came up with the modification of Minker's generalized closed world assumption that is particularly closely related to circumscription—the "extended" closed world assumption, or ECWA [Gelfond *et al.*, 1989]. The same idea is sometimes expressed in terms of "confirmation" and "unconfirmed formulas," or in terms of "characteristic clauses." Some of the other papers belonging to this line of research are [Bossu and Siegel, 1985], [Lifschitz, 1985], [Gelfond and Przymusinska, 1986], [Baker and Ginsberg, 1989], [Ginsberg, 1989], [Przymusinski, 1989], [Suchenek, 1989], [Gelfond *et al.*, 1990] and [Inoue and Helft, 1990].

The goal of this note is to show that the theorem from [Gelfond *et al.*, 1989] relating the ECWA to circumscription can be proved in a more general setting, and in a simpler way. The only thing that needs to be assumed in order to prove the the equivalence between circumscription and our reformulation of the ECWA is the very general condition known as "well-foundedness" [Etherington *et al.*, 1985], [Lifschitz, 1986].

2 Circumscription and well-founded sentences

In this note, a *sentence* is a sentence of a fixed second-order language. We understand *entailment* semantically (in view of the incompleteness of second-order logic, this is different from the deductive understanding of entailment). By P we denote a fixed tuple of distinct predicate constants, and Z stands for a fixed tuple of distinct object, function and/or predicate constants, disjoint from P. Recall that the circumscription operator transforms any sentence $A(P,Z)$ into the conjunction of $A(P,Z)$ and the minimality condition

$$\neg\exists pz(A(p,z) \wedge p < P) \tag{1}$$

([Genesereth and Nilsson, 1987], Chapter 6). We denote this conjunction by $A^c(P,Z)$.

A sentence $A(P,Z)$ is *well-founded* if it entails

$$\exists pz(A^c(p,z) \wedge p \leq P) \tag{2}$$

(see [Lifschitz, 1986], Proposition 2.1). It is easy to show, for instance, that every sentence that has no infinite models is well-founded. Other sufficient conditions for well-foundedness can be found in [Bossu and Siegel, 1985], [Etherington *et al.*, 1985], [Lifschitz, 1986] and [Etherington, 1988].

We will need the following property of well-founded sentences (for related results, see [Marczewski, 1951], [Lyndon, 1959], [Etherington *et al.*, 1985], [Etherington, 1988], [Gelfond *et al.*, 1989] and [Suchenek, 1990]).

Theorem 1. *Let A be a well-founded sentence, and let G be a sentence which contains no symbols from Z and no negative occurrences of symbols from P. If G is entailed by A^c, then it is entailed by A.*

Proof. Assume that $A(P,Z)$ is well-founded, that $G(P)$ contains no negative occurrences of symbols from P, and that $G(P)$ is entailed by

$A^c(P, Z)$. Then the sentence

$$A^c(P, Z) \supset G(P)$$

is universally valid. Consequently,

$$\forall pz(A^c(p, z) \supset G(p)) \tag{3}$$

is universally valid also. Obviously, (2) and (3) entail

$$\exists p(G(p) \wedge p \leq P). \tag{4}$$

Since $A(P, Z)$ is well-founded, we conclude that $A(P, Z)$ entails (4). It remains to notice that (4) entails $G(P)$, because all occurrences of the symbols from P in $G(P)$ are positive.

3 Main theorem

Theorem 2. *Let A be a well-founded sentence. A model of A satisfies A^c iff it satisfies every sentence F such that*

(i) F does not contain symbols from Z, and

(ii) for every sentence G which does not contain symbols from Z and does not contain negative occurrences of symbols from P, if G is entailed by $A \wedge F$, then G is entailed by A.

Proof. To prove the implication right to left, observe that the second conjunctive term of A^c, that is, (1), satisfies conditions (i) and (ii) as F. (For (ii), this fact follows from Theorem 1.) Let us prove the implication left to right. Consider any sentence $F(P)$ such that, for every sentence $G(P)$ which does not contain negative occurrences of symbols from P, if $G(P)$ is entailed by $A(P, Z) \wedge F(P)$, then $G(P)$ is entailed by $A(P, Z)$. We need to prove that $A^c(P, Z)$ entails $F(P)$. Take $G(P)$ to be

$$\exists pz(A(p, z) \wedge F(p) \wedge p \leq P). \tag{5}$$

Clearly, (5) is entailed by $A(P, Z) \wedge F(P)$; hence, by (ii), (5) is entailed by $A(P, Z)$. Assume $A^c(P, Z)$. Then $A(P, Z)$, and consequently (5). Take p, z such that

$$A(p, z) \wedge F(p) \wedge p \leq P.$$

Using the second conjunctive term of $A^c(P, Z)$, that is, (1), we derive from this formula $p = P$ and then $F(P)$.

4 Discussion

Let us compare Theorem 2 with the corresponding result from [Gelfond *et al.*, 1989].

The definition of the ECWA from Section 4 of that paper can be stated as follows. Let A be a first-order sentence without function symbols, which entails the Domain Closure Assumption

$$\forall x(x = c_1 \vee \ldots \vee x = c_n) \tag{6}$$

and the Unique Names Assumption

$$c_i \neq c_j \qquad (1 \leq i < j \leq n) \tag{7}$$

($c_1, \ldots, c_n$ are the object constants occurring in A). For any two disjoint lists P, Z of predicate constants from A, ECWA$(A; P; Z)$ stands for the first-order theory whose axioms are A and all sentences F such that

(i) F does not contain symbols from Z, and

(ii) for every disjunction G of ground literals which does not contain symbols from Z and does not contain negative occurrences of symbols from P, if G is entailed by A and F, then G is entailed by A.

In [Gelfond *et al.*, 1989], A is actually a set of first-order axioms, not necessarily finite, rather than a single sentence. It is clear, however, that, in the presence of (6), every sentence is equivalent to a propositional combination of ground atoms, so that an axiom set can include only finitely many nonequivalent axioms and consequently can be replaced by the conjunction of several of them. For this reason, we do not lose generality by assuming that A is a single sentence.

Corollary 4.7 from [Gelfond *et al.*, 1989] asserts that a structure satisfies A^c iff it is a model of ECWA$(A; P; Z)$. Theorem 2 differs from this result in two ways.

First, instead of (6) and (7), it assumes that A is well-founded. Since (6) implies that A has no infinite models, the assumption of well-foundedness is less restrictive.

Second, the version of condition (ii) used in the statement of Theorem 2 does not require that G be a disjunction of literals. It is easy to see, however, that, in the presence of (6), this difference is inessential. Indeed, any sentence G can be written as a conjunction of disjunctions of ground literals, and applying (ii) to a conjunction amounts to applying it to each conjunctive term separately.

This analysis shows that Corollary 4.7 from [Gelfond *et al.*, 1989] is subsumed by Theorem 2.

Acknowledgements

I am grateful to G. N. Kartha, Norman McCain and Marek Suchenek for comments on a draft of this note. This work was partially supported by National Science Foundation under grant IRI-9101078.

References

[Baker and Ginsberg, 1989] Andrew Baker and Matthew Ginsberg. A theorem prover for prioritized circumscription. In *Proc. of IJCAI-89*, pages 463–467, 1989.

[Bossu and Siegel, 1985] Genevieve Bossu and Pierre Siegel. Saturation, nonmonotonic reasoning, and the closed-world assumption. *Artificial Intelligence*, 25:13–63, 1985.

[Etherington *et al.*, 1985] David Etherington, Robert Mercer, and Raymond Reiter. On the adequacy of predicate circumscription for closed-world reasoning. *Computational Intelligence*, 1(1), 1985.

[Etherington, 1988] David Etherington. *Reasoning with Incomplete Information*. Pitman Publishing, London, 1988.

[Gelfond and Przymusinska, 1986] Michael Gelfond and Halina Przymusinska. Negation as failure: Careful closure procedure. *Artificial Intelligence*, 30(3):273–287, 1986.

[Gelfond *et al.*, 1989] Michael Gelfond, Halina Przymusinska, and Teodor Przymusinski. On the relationship between circumscription and negation as failure. *Artificial Intelligence*, 38(1):75–94, 1989.

[Gelfond *et al.*, 1990] Michael Gelfond, Halina Przymusinska, and Teodor Przymusinski. On the relationship between CWA, minimal model, and minimal Herbrand model semantics. *Int'l Journal of Intelligent Systems*, 5:549–564, 1990.

[Genesereth and Nilsson, 1987] Michael R. Genesereth and Nils J. Nilsson. *Logical Foundations of Artificial Intelligence.* Morgan Kaufmann, San Mateo, CA, 1987.

[Ginsberg, 1989] Matthew Ginsberg. A circumscriptive theorem prover. *Artificial Intelligence*, 39:209–230, 1989.

[Inoue and Helft, 1990] Katsumi Inoue and Nicolas Helft. On theorem provers for circumscription. In *Proc. of CSCSI-90*, pages 212–219, 1990.

[Lifschitz, 1985] Vladimir Lifschitz. Closed-world data bases and circumscription. *Artificial Intelligence*, 27, 1985.

[Lifschitz, 1986] Vladimir Lifschitz. On the satisfiability of circumscription. *Artificial Intelligence*, 28, 1986.

[Lyndon, 1959] Roger Lyndon. Properties preserved under homomorphism. *Pacific Journal of Mathematics*, 9:143–154, 1959.

[Marczewski, 1951] Edward Marczewski. Sur les congruences et les prorietés positives d'algébres abstraites. *Colloquium Mathematicum*, 2:220, 1951.

[McCarthy, 1980] John McCarthy. Circumscription—a form of non-monotonic reasoning. *Artificial Intelligence*, 13:27–39,171–172, 1980. Reproduced in [McCarthy, 1990].

[McCarthy, 1990] John McCarthy. *Formalizing common sense: papers by John McCarthy.* Ablex, Norwood, NJ, 1990.

[Minker, 1982] Jack Minker. On indefinite data bases and the closed world assumption. In *Proc. of CADE-82*, pages 292–308, 1982.

[Minker, 1988] Jack Minker. Perspectives in deductive databases. *Journal of Logic Programming*, 5:33–60, 1988.

[Przymusinski, 1989] Teodor Przymusinski. An algorithm to compute circumscription. *Artificial Intelligence*, 38(1):49–73, 1989.

[Reiter, 1978] Raymond Reiter. On closed world data bases. In Herve Gallaire and Jack Minker, editors, *Logic and Data Bases*, pages 119–140. Plenum Press, New York, 1978.

[Suchenek, 1989] Marek Suchenek. A syntactic characterization of minimal entailment. In Ewing Lusk and Ross Overbeek, editors, *Logic programming: North American Conference*, pages 81–91, 1989.

[Suchenek, 1990] Marek Suchenek. Applications of Lyndon homomorphism theorems to the theory of minimal models. *Int'l Journal of Foundations of Computer Science*, 1:49–59, 1990.