A Causal Theory of Ramifications and Qualifications

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Abstract

This paper is concerned with the problem of determining the indirect effects or *ramifications* of actions. We argue that the standard framework in which background knowledge is given in the form of state constraints is inadequate and that background knowledge should instead be given in the form of "causal laws." We represent "causal laws" first as inference rules and later as sentences in a modal, conditional logic $\mathcal{C}_{\text{flat}}$. For the framework with "causal laws," we propose a simple fixpoint condition defining the possible next states after performing an action. This fixpoint condition guarantees minimal change between states, but also enforces the requirement that changes be "caused." Ramification and qualification constraints can be expressed as "causal laws."

1 Introduction

This paper is concerned with the problem of determining the indirect effects or ramifications of actions. The problem is usually investigated, as in [Kartha and Lifschitz, 1994], in a framework in which action domains are described in part by state constraints. (Informally, a state constraint is a formula that says of a proposition that it is true in every possible state of the world.) Our main objective is to argue that an adequate theory of ramifications requires the representation of information of a kind that is not conveyed by state constraints. In particular, what is required is the representation of the causal relations (or, more generally, the determination relations) that hold between states of affairs. It turns out that this is also the information that is needed for an adequate theory of derived action preconditions or qualifications.

Previous approaches to the problem of ramifications have assumed a definition of the following kind: A ramification, roughly speaking, is a change (not explicitly described) that is implied by the performance of an action. In our approach, we substitute the word "caused" for the word "implied." In determining the ramifications of actions, it is not enough, we say, to infer that a change must occur when an action is performed; it is necessary to infer that the action causes the change to occur. As we will see, this stronger requirement makes it possible to avoid unintended ramifications and to infer derived qualifications. (The need for the latter is argued in [Ginsberg and Smith, 1988] and [Lin and Reiter, 1994].) Again roughly speaking, our theory of qualifications is this: An action cannot be performed if the performance of the action implies a change that it does not cause.[1]

The main points can be illustrated by the following example. Imagine that Fred the turkey is on a walk. Consider the action of making Fred dead. Intuitively, as an indirect effect of performing the action, Fred will no longer be walking. The reason is that Fred's being dead causes him to stop walking. Now consider the action of making Fred walk, but suppose that Fred is dead. Intuitively, the action cannot be performed. The reason is as follows: Fred can walk only if he is alive, but making him walk does not cause him to be alive; so unless he is already alive (or something in addition is done to cause him to become alive) he cannot be made to walk.

The conclusions reached in the previous paragraph are supported by the following facts. Intuitively, Fred can be made to not walk *by* making him not alive, but he cannot be made to be alive *by* making him walk. If the indirect effects of an action are the facts made true *by* making the direct effects of the action true, then we should expect Fred's not walking to be an indirect effect of making him not alive, but we should not expect Fred's being alive to be an indirect effect of making him walk.

In general, facts about what can and cannot be done *by* doing something else are contingent upon underlying causal connections and other relations of determination. (For a discussion of noncausal determination relations see [Kim, 1974].) State constraints say nothing about these, so it is not surprising that background knowledge in the form of state constraints should prove to be inadequate. In the recent literature on reasoning about

[*]Partially supported by National Science Foundation grant #IRI-9306751.

[†]Partially supported by an IBM Graduate Fellowship and by NSF grant #IRI-9306751.

[1]A similar proposal by Lin [1995] appears in these proceedings.

action, the inadequacy of state constraints has been observed by Elkan [1992] and by Brewka and Hertzberg [1993].

The central problem addressed in this paper is that of properly defining the set of possible next states after performing an action, given specific direct effects and background knowledge in the form of "causal laws."[2]

For the standard framework in which background knowledge is given in the form of state constraints, this problem was solved by Winslett [1988]. It is not clear, however, how Winslett's definition should be modified to accommodate causal laws. Accordingly, after an initial discussion of notation and terminology in Section 2, we go on in Section 3 to reformulate Winslett's definition, obtaining an equivalent definition of a quite different form. On the basis of this reformulation, in Section 4 we present our definition for the framework with "causal laws," represented as inference rules. In Section 5, we show how ramification and qualification constraints can be encoded by inference rules. We also show that the framework with inference rules properly extends the standard framework.

In Sections 6 and 7, we give a semantic account of "causal laws," replacing inference rules by a rule-like conditional, for which a modal, partial state semantics is defined. Doing so allows us to clarify the sense in which our theory of ramifications and qualifications is a causal theory.

2 Notation and Terminology

We begin with a standard language of propositional logic. We represent an interpretation for the language by a maximal consistent set of literals. (Throughout this paper, we use the symbol L to stand exclusively for literals.) Informally, we think of an interpretation as a logically possible state of the world. For convenience, we sometimes use the word "state" to mean an interpretation. By an *explicit effect* we mean a set of formulas. Intuitively, these are the formulas that an action is explicitly said to cause.

The central problem in determining the ramifications of an action is to properly define $Res(E, S)$, the set of possible next states after performing an action with the explicit effect E in the state S. We address this problem in frameworks in which background knowledge is given in the form of state constraints and "causal laws."

By a *formula constraint* we mean a formula. We will require every given formula constraint to hold in every possible next state, so a formula constraint functions (in this respect) as a state constraint. A standard example of a formula constraint is the formula $(Walking \supset Alive)$ [Baker, 1991].

We will write an inference rule as an expression of the form

$$\phi \Rightarrow \psi \tag{1}$$

[2]Throughout this paper, we use the term "causal law" in place of the more accurate but less familiar term "determination relation." Causal laws are in any case prime examples of determination relations.

where ϕ and ψ are formulas. Informally, we will think of (1) as expressing a relation of determination between the states of affairs that make ϕ and ψ true. One kind of determination relation is causal. As an example, we think of the rule $\neg Alive \Rightarrow \neg Walking$ as expressing the "causal law" that not being alive causes not walking. Of course, walking does not similarly cause being alive, which shows that the causal relation, like the inference rule, is "noncontrapositive."

The standard derivability relation $\vdash$ of propositional logic is easily extended to take account of inference rules. Given a set Γ of formulas, a set C of inference rules, and a formula ϕ, we write

$$\Gamma \vdash_C \phi$$

to mean that ϕ is an element of the smallest set of formulas that contains Γ and is closed under C and propositional logic.

3 Possible Next States: Constraints

The standard framework in which the problem of ramifications is addressed is one in which background knowledge is given in the form of state constraints. For this framework, the problem was solved by Winslett [1988].

Definition W [Winslett, 1988]. For all interpretations S and S', any explicit effect E, and any set B of formula constraints, $S' \in Res^W_B(E, S)$ if

(1) S' satisfies $E \cup B$, and

(2) no other interpretation that satisfies $E \cup B$ differs from S on fewer atoms, where "fewer" is defined by set inclusion.

Intuitively, $Res^W_B(E, S)$ is the set of possible next states after performing an action with the explicit effect E in the state S, given background knowledge B.

As an example, let

$$\begin{aligned} S &= \{Alive, Walking\} \\ E &= \{\neg Alive\} \\ B &= \{Walking \supset Alive\}. \end{aligned}$$

Then

$$Res^W_B(E, S) = \{\{\neg Alive, \neg Walking\}\}.$$

Here, $\neg Walking$ is a ramification.

It is not clear how to modify Winslett's definition to accommodate background knowledge in the form of inference rules. Accordingly, the remainder of this section will be devoted to reformulating it.

In order to explain our reformulation, we present a series of three definitions, in which the assumption of inertia and background knowledge are introduced in successive steps. Each definition will take the following form: For all interpretations S and S', and any explicit effect E, $S' \in Res(E, S)$ if S' is precisely the set of literals that are derivable from E and the available background information (possibly including information provided by the assumption of inertia). Recall that throughout this paper the symbol L will be used to stand exclusively for literals.

In Definition 1, we do not assume the principle of inertia, nor do we include formula constraints. Consequently, every literal in any possible next state must be derivable from E alone.

Definition 1. For all interpretations S and S', and any explicit effect E, $S' \in \mathit{Res}^1(E,S)$ if

$$S' = \{L : E \vdash L\}.$$

Clearly, if the set $\{L : E \vdash L\}$ is an interpretation then this interpretation is the only element of $\mathit{Res}^1(E,S)$. Otherwise, either E does not imply a value for every atom or E is inconsistent. Since E is required to specify not only the values of the atoms that change but also the values of those that do not, the frame problem is unsolved in Definition 1.

Consider, for example, the state $S = \{p, q\}$ and an action that makes p false. Choosing $E = \{\neg p \wedge q\}$, we have $\mathit{Res}^1(E,S) = \{\{\neg p, q\}\}$.

In the next definition (Definition 2), we assume the principle of inertia, but do not yet include formula constraints. Here the frame problem is solved by the principle of inertia, which makes it possible to specify only the values of the atoms that change. The values of the other atoms are assumed to remain the same in S' as in S. The literals in $S \cap S'$ are those whose values are preserved by inertia. We obtain Definition 2 by adding the literals in $S \cap S'$ as additional premises to the derivability condition in Definition 1.

Definition 2. For all interpretations S and S', and any explicit effect E, $S' \in \mathit{Res}^2(E,S)$ if

$$S' = \{L : (S \cap S') \cup E \vdash L\}.$$

Consider again the state $S = \{p, q\}$ and an action that makes p false. Choosing $E = \{\neg p\}$, we have $\mathit{Res}^2(E,S) = \{\{\neg p, q\}\}$. Because of the assumption of inertia, $\mathit{Res}^2(E,S)$ may be nonempty even when the explicit effect does not imply a value for every atom.

The solutions to the equation in Definition 2 are the interpretations S' that are fixpoints of the function $\lambda X.\{L : (S \cap X) \cup E \vdash L\}$. The following example shows that there may be more than one fixpoint. Let

$$\begin{array}{rcl} S & = & \{\neg p, \neg q\} \\ E & = & \{p \vee q\}. \end{array}$$

Then

$$\mathit{Res}^2(E,S) = \{\{\neg p, q\}, \{p, \neg q\}\}.$$

The definition of the transition function for the language $\mathcal{A}$ of Gelfond and Lifschitz [1993] corresponds to the special case of Definition 2 in which the explicit effect E is required to be a consistent set of literals. Under this restriction, it is easy to see that $\mathit{Res}^2(E,S)$ will always be a singleton.

We are now ready to reformulate Winslett's definition for the framework with inertia and background knowledge in the form of formula constraints. We obtain Definition 3 by simply adding B to the premises of the derivability condition in Definition 2.

Definition 3. For all interpretations S and S', any explicit effect E, and any set B of formula constraints, $S' \in \mathit{Res}^3_B(E,S)$ if

$$S' = \{L : (S \cap S') \cup E \cup B \vdash L\}.$$

Winslett's definition expresses the idea of minimizing change. Definition 3 has a very different form; it is given in terms of a fixpoint condition. Despite this difference, the two definitions are equivalent, as the following proposition shows.

Proposition 1 *For any interpretation S, any explicit effect E, and any set B of formula constraints, $\mathit{Res}^W_B(E,S) = \mathit{Res}^3_B(E,S)$.*

Proof. For the left-to-right direction, assume $S' \in \mathit{Res}^W_B(E,S)$. Suppose S'' is a model of $(S \cap S') \cup E \cup B$. It follows that $S \cap S' \subseteq S''$, so $S \cap S' \subseteq S \cap S''$. Since $S' \in \mathit{Res}^W_B(E,S)$ and S'' is a model of $E \cup B$, we know that $S \cap S'$ is not a proper subset of $S \cap S''$. Thus, $S \cap S' = S \cap S''$. It follows that $S'' = S'$, since S, S', and S'' are interpretations. So S' is the unique model of $(S \cap S') \cup E \cup B$. Therefore, $S' = \{L : (S \cap S') \cup E \cup B \vdash L\}$, and so $S' \in \mathit{Res}^3_B(E,S)$. For the right-to-left direction, assume that $S' \in \mathit{Res}^3_B(E,S)$. Then $S' = \{L : (S \cap S') \cup E \cup B \vdash L\}$. It follows that S' is the unique model of $(S \cap S') \cup E \cup B$. Let S'' be a member of $\mathit{Res}^W_B(E,S)$ such that $S \cap S' \subseteq S \cap S''$. We will show that $S' = S''$. We know that S'' is a model of $E \cup B$. Since $S \cap S' \subseteq S \cap S''$, S'' is a model of $S \cap S'$. So S'' is a model of $(S \cap S') \cup E \cup B$. Therefore, $S'' = S'$. □

4 Possible Next States: Causal Laws

Given our reformulation of Winslett's definition, it is now a simple matter to define the possible next states in the presence of background knowledge in the form of "causal laws," represented as inference rules.

We obtain the definition for Res in the present framework by replacing $\vdash$ in Definition 2 by $\vdash_C$. We do not include formula constraints (by similarly modifying Definition 3), because, as we will see, for the purpose of defining Res they are easily represented by inference rules.[3]

Definition 4. For all interpretations S and S', any explicit effect E, and any set C of inference rules, $S' \in \mathit{Res}^4_C(E,S)$ if

$$S' = \{L : (S \cap S') \cup E \vdash_C L\}.$$

For example, let

$$\begin{array}{rcl} S & = & \{\mathit{Alive}, \mathit{Walking}\} \\ E & = & \{\neg \mathit{Alive}\} \\ C & = & \{\neg \mathit{Alive} \Rightarrow \neg \mathit{Walking}\}. \end{array}$$

Then

$$\mathit{Res}^4_C(E,S) = \{\{\neg \mathit{Alive}, \neg \mathit{Walking}\}\}.$$

Again, $\neg \mathit{Walking}$ is a ramification.

[3] Definition 4 is closely related to the definition of "rule update" in [Przymusinski and Turner, 1995], which generalizes revision programming [Marek and Truszczyński, 1994].

One advantage of Definition 4 over Definition 3 is illustrated by the following variation on the previous example. Let

$$\begin{array}{rcl} S & = & \{\neg Alive, \neg Walking\} \\ E & = & \{Walking\} \\ B & = & \{Walking \supset Alive\} \end{array}$$

with C as before. Then $Res^4_C(E,S)$ is empty, whereas

$$Res^3_B(E,S) = \{\{Alive, Walking\}\}.$$

Intuitively, $Res^4_C(E,S)$ is correct. Since we cannot make *Alive* true by making *Walking* true, we cannot perform an action in state S whose explicit effect is $\{Walking\}$, because this effect implies a change (namely, making *Alive* true) that the action does not cause.[4] This is an example of a derived qualification.

Another advantage of using causal laws can be illustrated using the domain introduced in [Lifschitz, 1990] in which there are two switches and a light. Let

$$\begin{array}{rcl} S & = & \{\neg Up1, Up2, \neg On\} \\ E & = & \{Up1\} \\ B & = & \{On \equiv (Up1 \equiv Up2)\}. \end{array}$$

Then $Res^3_B(E,S)$ contains two states: $\{Up1, Up2, On\}$ and $\{Up1, \neg Up2, \neg On\}$. The second state in $Res^3_B(E,S)$ is anomalous, and results from the unintended ramification $\neg Up2$. In [Lifschitz, 1990] and [Kartha and Lifschitz, 1994], this ramification is blocked by declaring *Up1* and *Up2* to be "in the frame" and *On* to be "not in the frame." By constrast, the use of inference rules in place of formula constraints makes the frame/nonframe distinction unnecessary for the purpose of limiting possible ramifications. For instance, let C contain the inference rules

$$(Up1 \equiv Up2) \Rightarrow On$$
$$\neg(Up1 \equiv Up2) \Rightarrow \neg On.$$

Then

$$Res^4_C(E,S) = \{\{Up1, Up2, On\}\}.$$

Notice that in the previous examples, $Res^4_C(E,S)$ is a subset of $Res^3_B(E,S)$. The following proposition shows that this relationship holds whenever B and C are related as above.

Proposition 2 *Let C be a set of inference rules, and let $B = \{\phi \supset \psi : \phi \Rightarrow \psi \in C\}$. For any interpretation S and explicit effect E, $Res^4_C(E,S)$ is a subset of $Res^3_B(E,S)$.*

Proof. Suppose that $S' \in Res^4_C(E,S)$. So $S' = \{L : (S \cap S') \cup E \vdash_C L\}$. It follows that $S' = \{L : S' \vdash_C L\}$. Therefore, for any formula ϕ, $S' \vdash_C \phi$ if and only if $S' \models \phi$. Now consider any formula $\phi \supset \psi$ in B such that $S' \models \phi$. We know that $S' \vdash_C \phi$, and since $\phi \Rightarrow \psi$ belongs to C, it follows that $S' \vdash_C \psi$. Therefore $S' \models \psi$, which shows that S' is a model of B. Clearly, S' also satisfies E, so we have shown that S' satisfies $E \cup B$. Let S'' be a model of $E \cup B$ such that $S \cap S' \subseteq S \cap S''$. We need to show that $S'' = S'$. Since S' and S'' are interpretations, it is enough to show that $S' \subseteq S''$.

$$\begin{array}{rlr} S' & = \{L : (S \cap S') \cup E \vdash_C L\} & \{S' \in Res^4_C(E,S)\} \\ & \subseteq \{L : (S \cap S'') \cup E \vdash_C L\} & \{S \cap S' \subseteq S \cap S''\} \\ & \subseteq \{L : S'' \cup E \vdash_C L\} & \{S \cap S'' \subseteq S''\} \\ & = \{L : S'' \vdash_C L\} & \{S'' \text{ satisfies } E\} \end{array}$$

Since $B = \{\phi \supset \psi : \phi \Rightarrow \psi \in C\}$, we have $\{L : S'' \vdash_C L\} \subseteq \{L : S'' \cup B \vdash L\}$. Finally, since S'' satisfies B, $\{L : S'' \cup B \vdash L\} = S''$. □

5 Ramification and Qualification Constraints

Lin and Reiter [1994] draw a pragmatic distinction between two kinds of state constraints: *ramification constraints*, which yield indirect effects, and *qualification constraints*, which yield action preconditions. As they observe, the same distinction was drawn earlier by Ginsberg and Smith [1988]. In the language of inference rules, we can give a syntactic form to this distinction. Suppose that ϕ is a formula constraint. If we wish ϕ to function as a ramification constraint, we write the rule

$$True \Rightarrow \phi.$$

If instead we wish ϕ to function as a qualification constraint we write the rule

$$\neg\phi \Rightarrow False.$$

In Definition 3 all formula constraints function as ramification constraints. The correctness of our encoding of ramification constraints is demonstrated by the following proposition.

Proposition 3 *Let B be a set of formula constraints. Let C be the set of inference rules $\{True \Rightarrow \phi : \phi \in B\}$. For every interpretation S and explicit effect E, $Res^3_B(E,S) = Res^4_C(E,S)$.*

Proof. Due to the special form of the rules in C, it is clear that for any set Γ of formulas and any formula ϕ, we have $\Gamma \cup B \vdash \phi$ if and only if $\Gamma \vdash_C \phi$. Hence, $S' \in Res^3_B(E,S)$ iff $S' = \{L : (S \cap S') \cup E \cup B \vdash L\}$ iff $S' = \{L : (S \cap S') \cup E \vdash_C L\}$ iff $S' \in Res^4_C(E,S)$. □

The preceding proposition also shows that the framework of Definition 3 is subsumed by that of Definition 4.

As an example of a domain in which a state constraint is intended to function as a qualification constraint, we consider a simplified version of a domain from [Lin and Reiter, 1994]. Imagine an ancient kingdom in which there are two blocks. Either block may be painted yellow, but by order of the emperor at most one of the blocks is permitted to be yellow at a time. Consider a state in which the second block is yellow. Intuitively, in

[4] Intuitively, since not being alive causes not walking, the conditional $(Walking \supset Alive)$ holds in every possible state of the world, and in this sense *Walking* implies *Alive*. However, the inference rule $(\neg Alive \Rightarrow \neg Walking)$ does not capture this intuition. This is a deficiency in the representation of causal laws which is remedied in Sections 6 and 7.

this state it is not possible to paint the first block yellow. Representing the emperor's decree by a ramification constraint does not conform to this intuition. Let

$$\begin{aligned} S &= \{\neg Y_1, Y_2\} \\ E &= \{Y_1\} \\ C &= \{\mathit{True} \Rightarrow \neg(Y_1 \wedge Y_2)\}. \end{aligned}$$

Then

$$\mathit{Res}^4_C(E,S) = \{\{Y_1, \neg Y_2\}\}.$$

So painting the first block yellow changes the color of the second block! On the other hand, if we represent the emperor's decree as a qualification constraint by redefining C as

$$C = \{(Y_1 \wedge Y_2) \Rightarrow \mathit{False}\}$$

then $\mathit{Res}^4_C(E,S)$ is empty, which conforms to our intuition that it is impossible to paint the first block yellow in state S.

The following straightforward proposition shows that rules of the form we write for qualification constraints cannot lead to ramifications, but can only rule them out.

Proposition 4 *Let C be a set of inference rules and ϕ be a formula. Let $C' = C \cup \{\neg\phi \Rightarrow \mathit{False}\}$. For all interpretations S and S', and any explicit effect E, $S' \in \mathit{Res}^4_{C'}(E,S)$ iff $S' \in \mathit{Res}^4_C(E,S)$ and $S' \models \phi$.*

Brewka and Hertzberg [1993], who also use inference rules to represent causal laws, propose a modification of Winslett's definition [1988] in which causal laws play a role in the definition of minimal change between states. Because of the role that minimal change continues to play in their definition, they cannot express qualification constraints in manner shown above. Nor do they obtain derived qualifications of the kind illustrated by the *Walking*/*Alive* example of Section 4.[5] Notice in that example that the causal law $\neg \mathit{Alive} \Rightarrow \neg \mathit{Walking}$ is neither a ramification constraint nor a qualification constraint, but rather functions in both ways, depending upon the explicit effect.

6 The Logic of S-Conditionals

Representing "causal laws" by inference rules is convenient for the purpose of exposition, but it is not ultimately satisfactory, for two reasons. First, an inference rule is not an object language sentence, and therefore, unlike, for example, a formula constraint, is not interpreted declaratively. As a result, whereas Definitions 1–3 of Section 3 may be recast in semantic terms by simply replacing the derivability relation $\vdash$ by the consequence relation $\models$, this is not true of Definition 4. Secondly, although a causal law intuitively implies its corresponding material conditional (indeed, it implies the corresponding strict conditional, which says that the material conditional holds in every possible state of the world), an inference rule does not. In this section, we prepare to remedy these deficiencies in the representation of "causal laws" by defining a new conditional logic $\mathcal{C}_{\text{flat}}$, which is an extension of S5 modal logic.

The language $\mathcal{C}_{\text{flat}}$ is obtained by adding the following clause to the usual inductive definition of formulas of modal logic: if ϕ and ψ are formulas of propositional logic then

$$(\phi \Rightarrow \psi) \qquad (2)$$

is a formula (of $\mathcal{C}_{\text{flat}}$).[6] The formula (2) is called an *s-conditional*. Intuitively, (2) may be read as: the truth of ϕ determines the truth of ψ.[7] Note that s-conditionals have the same syntactic form that we have used for inference rules. Informally, $\phi \Rightarrow \psi$ is true just in case in every part of every possible state of the world in which ϕ is true, ψ is true as well.

An S5 structure can be defined as a pair (Σ, S), where Σ is a nonempty set of interpretations, and S is a distinguished interpretation in Σ. A $\mathcal{C}_{\text{flat}}$ structure is obtained by replacing the set Σ by a set Ω of sets of interpretations.

A structure for a specific $\mathcal{C}_{\text{flat}}$ language $\mathcal{L}$ is a pair (Ω, S), where Ω is a nonempty set of nonempty sets of interpretations of $\mathcal{L}$, and S is an interpretation of $\mathcal{L}$ such that $\{S\} \in \Omega$. The elements of Ω are called *partial states*. By $\mathcal{S}(\Omega)$ we designate the set of *states*, defined as: $\mathcal{S}(\Omega) = \{S : \{S\} \in \Omega\}$. (The set $\mathcal{S}(\Omega)$ corresponds to the set Σ in an S5 structure.) We impose the following structure condition: for every partial state $U \in \Omega$, there is a state $S \in \mathcal{S}(\Omega)$ such that $S \in U$. This reflects the natural requirement that every partial state be a part of some state.

For any set U of interpretations and any formula ϕ of propositional logic, we write $U \models \phi$ as an abbreviation for the expression: for all $S \in U$, $S \models \phi$. Furthermore, for any set Γ of formulas of propositional logic, we write $U \models \Gamma$ as an abbreviation for the expression: for all $\phi \in \Gamma, U \models \phi$.

We define when a structure (Ω, S) *satisfies* a formula ϕ (in symbols, $(\Omega, S) \models \phi$) as follows. For all formulas ϕ and ψ (except in the last clause below),

$$\begin{aligned} &(\Omega,S) \models \phi \text{ iff } \phi \in S, \quad \text{if } \phi \text{ is an atom}, \\ &(\Omega,S) \models \neg\phi \text{ iff } (\Omega,S) \not\models \phi, \\ &(\Omega,S) \models \phi \wedge \psi \text{ iff } (\Omega,S) \models \phi \text{ and } (\Omega,S) \models \psi, \\ &(\Omega,S) \models \Box\phi \text{ iff for all } S \in \mathcal{S}(\Omega),\ (\Omega,S) \models \phi, \\ &(\Omega,S) \models \phi \Rightarrow \psi \text{ iff} \\ &\qquad \text{for all } U \in \Omega, \text{ if } U \models \phi \text{ then } U \models \psi\,. \end{aligned}$$

In the last clause, ϕ and ψ are formulas of propositional logic.

[5] Even in cases where derived qualifications are not involved, Brewka and Hertzberg may obtain results different from those of Definition 4.

[6] Requiring ϕ and ψ to be formulas of propositional logic is not essential. In fact, the language $\mathcal{C}_{\text{flat}}$ is a simplified version of a more general language $\mathcal{C}$, in which ϕ and ψ are permitted to be arbitrary formulas, possibly containing the modal operators $\Box$ and $\Diamond$, or $\Rightarrow$. The advantage of $\mathcal{C}_{\text{flat}}$ is that it is possible to give its semantics using a simpler kind of model structure than is required for $\mathcal{C}$.

[7] An alternative reading is: ϕ's being true is a sufficient condition for ψ's being true. This reading is our motivation for the name "s-conditional."

Let T be a set of formulas. A *model* of T is a structure that satisfies every formula in T. We say that T *entails* a formula ϕ (in symbols, $T \models \phi$) if every model of T is a model of ϕ.

It is easy to see that an s-conditional entails its corresponding strict conditional, that is,

$$\phi \Rightarrow \psi \models \Box(\phi \supset \psi). \tag{3}$$

It is also easy to see that an s-conditional does not necessarily entail its contrapositive. For example, let $S_1 = \{a, b\}$, $S_2 = \{a, \neg b\}$, $S_3 = \{\neg a, \neg b\}$, and

$$M = (\{\{S_1\}, \{S_3\}, \{S_2, S_3\}\}, S_1)\,.$$

Observe that $M \models a \Rightarrow b$, but $M \not\models \neg b \Rightarrow \neg a$. These are two important properties of causal laws.

6.1 S-Conditionals and Inference Rules

In this subsection we briefly investigate the relationship between s-conditionals and inference rules. Throughout this subsection we will use the term formula to mean a formula of propositional logic.

For any set U of interpretations, by $Th(U)$ we mean the set of formulas ϕ such that $U \models \phi$. For any set Γ of formulas, by $Mod(\Gamma)$ we mean the set of interpretations that satisfy Γ.

Fact 1. For any set U of interpretations and any formula ϕ, $U \models \phi$ if and only if $\phi \in Th(U)$.

Fact 2. Let C be a set of expressions having the common form of s-conditionals and inference rules. Let (Ω, S) be a model of C. If $U \in \Omega$ then $Th(U)$ is closed under C.

Given a set C of s-conditionals, a model (Ω, S) of C is called *maximal* if there is no model (Ω', S') of C such that Ω is a proper subset of Ω'. It is clear that the maximal models of C can differ only in their second component.

Even when (Ω, S) is maximal, the converse of Fact 2 may fail to hold. This is due to the structure condition which requires every set $U \in \Omega$ to contain an interpretation S' such that $\{S'\} \in \Omega$. This complication motivates the following definition.

Let Γ be a set of formulas, and let C be a set of inference rules. We say that Γ is *completable wrt C* if there exists a maximal consistent superset of Γ that is closed under C.

Fact 3. Let C be a set of expressions having the common form of s-conditionals and inference rules. Let (Ω, S) be a maximal model of C. For any set U of interpretations, if $Th(U)$ is closed under C and completable wrt C then $U \in \Omega$.

In the next section we will use the following lemma to justify a reformulation of Definition 4.

Lemma 1 *Let C be a set of expressions having the common form of s-conditionals and inference rules. Let (Ω, S) be a maximal model of C. For any completable set Γ of formulas and any formula ϕ, $\Gamma \vdash_C \phi$ if and only if for all $U \in \Omega$, if $U \models \Gamma$ then $U \models \phi$.*

Proof. For the left-to-right direction, assume that $\Gamma \vdash_C \phi$. Suppose $U \in \Omega$ and $U \models \Gamma$. Then $Th(U)$ is logically closed, closed under C (by Fact 2), and contains Γ. Since $\Gamma \vdash_C \phi$, we know that ϕ is in the smallest such set. So $\phi \in Th(U)$. By Fact 1, $U \models \phi$.

For the right-to-left direction, assume that for all $U \in \Omega$, if $U \models \Gamma$ then $U \models \phi$. Let Γ' be the smallest logically closed set that is closed under C and contains Γ. We will show that Γ' is completable wrt C. Since Γ is completable wrt C, we know there is a maximal consistent superset Γ'' of Γ that is closed under C. Thus, Γ'' is a logically closed set that is closed under C and contains Γ. Since Γ' is the smallest such set, $\Gamma' \subseteq \Gamma''$. Hence, Γ' is completable wrt C. Notice that $\Gamma' = Th(Mod(\Gamma'))$. Thus, by Fact 3, $Mod(\Gamma') \in \Omega$. Moreover, since $\Gamma \subseteq \Gamma'$, $Mod(\Gamma') \models \Gamma$. Since $Mod(\Gamma') \in \Omega$, $Mod(\Gamma') \models \phi$. Thus, by Fact 1, $\phi \in \Gamma'$. By the choice of Γ', $\Gamma \vdash_C \phi$. □

7 Causality versus Implication

In this section, we recast Definition 4 semantically, representing causal laws as s-conditionals, instead of as inference rules. Doing so will allow us to clarify the sense in which our theory of ramifications and qualifications is a "causal" theory.

Definition 4′. Let C be a set of s-conditionals, and let $M = (\Omega, S'')$ be a maximal model of C. For all states $S, S' \in \mathcal{S}(\Omega)$ and any explicit effect E, $S' \in Res^{4'}_M(E, S)$ if

$$S' = \left\{ L : \begin{array}{l} \textit{for all } U \in \Omega, \\ \textit{if } U \models (S \cap S') \cup E \textit{ then } U \models L \end{array} \right\}.$$

The following proposition shows that Definitions 4 and 4′ agree where both are defined.

Proposition 5 *Let C be a set of expressions having the common form of s-conditionals and inference rules. Let $M = (\Omega, S')$ be a maximal model of C. For all states $S \in \mathcal{S}(\Omega)$ and any explicit effect E, $Res^4_C(E, S) = Res^{4'}_M(E, S)$.*

Proof. For any set Γ of formulas, let $\lambda(\Gamma) = \{L :$ for all $U \in \Omega$, if $U \models \Gamma$ then $U \models L\}$. We will show that for any interpretation S, $S = \{L : \Gamma \vdash_C L\}$ iff $S = \lambda(\Gamma)$.

For the left-to-right direction, let S be an interpretation and Γ be a set of formulas such that $S = \{L : \Gamma \vdash_C L\}$. Because $S = \{L : \Gamma \vdash_C L\}$ and S is an interpretation, $Th(\{S\}) = \{\phi : \Gamma \vdash_C \phi\}$. Clearly, $Th(\{S\})$ contains Γ and is closed under C, which shows that Γ is completable wrt C. By Lemma 1, $\{L : \Gamma \vdash_C L\} = \lambda(\Gamma)$. So $S = \lambda(\Gamma)$.

For the right-to-left direction, let S be an interpretation and Γ a set of formulas such that $S = \lambda(\Gamma)$. Since $\lambda(\Gamma)$ is consistent, we know there is a $U \in \Omega$ that satisfies Γ. Moreover, since $\lambda(\Gamma) = S$, we know that for every such U, $U \models L$ for every $L \in S$. This shows that $\{S\}$ belongs to Ω and satisfies Γ. By Fact 1, since $\{S\} \models \Gamma$, $\Gamma \subseteq Th(\{S\})$. By Fact 2, since $\{S\} \in \Omega$, $Th(\{S\})$ is closed under C. So Γ is completable wrt C. By Lemma 1, $\{L : \Gamma \vdash_C L\} = \lambda(\Gamma)$. So $S = \{L : \Gamma \vdash_C L\}$. □

Given Propositions 3 and 4, it follows from Proposition 5 that s-conditionals of the forms $True \Rightarrow \phi$ and $\neg\phi \Rightarrow False$ function as ramification and qualification constraints, respectively. It is interesting to note that the first of these says that ϕ holds in every partial state, whereas the second says that $\neg\phi$ holds in no partial state. Because partial states have truth value gaps, these meanings are distinct. Thus, ramification and qualification constraints are now distinguished semantically as well as syntactically.

In order to further explain what is distinctive about our theory, it will be convenient to assume, for the time being, that $(S \cap S') \cup E$ is finite. Moreover, in order to simplify notation, whenever $(S \cap S') \cup E$ appears in a formula we will take it to stand for the (finite) conjunction of its elements. Using this convention, we can say that $S' \in Res^{4'}_M(E, S)$ if and only if

$$S' = \{L : M \models (S \cap S') \cup E \Rightarrow L\}. \qquad (4)$$

Given an $S' \in Res^{4'}_M(E, S)$, we will say that a literal $L \in S'$ is a ramification if $(S \cap S') \cup E \not\models L$. Assume that L is such a ramification. It is interesting to contrast the condition

$$M \models (S \cap S') \cup E \Rightarrow L$$

from Equation (4), which roughly speaking says that L is caused, with a similar condition

$$M \models \Box\,((S \cap S') \cup E \supset L)$$

which says that L is implied, in the sense that in every possible state in which $(S \cap S') \cup E$ is true, L is true. Since, by (3), $(S \cap S') \cup E \Rightarrow L$ entails $\Box\,((S \cap S') \cup E \supset L)$, it is clear that the former condition is stronger than the latter.

If we wished to require merely that ramifications be implied, rather than caused, we would instead impose the fixpoint condition

$$S' = \{L : M \models \Box\,((S \cap S') \cup E \supset L)\}. \qquad (5)$$

This condition corresponds to Definition 3. To see this, let B be a set of formula constraints, $\Box B = \{\Box\phi : \phi \in B\}$, and M be a maximal $\mathcal{C}_{\text{flat}}$ model of $\Box B$.[8] It can be shown that

$$(S \cap S') \cup E \cup B \vdash L \;\; \textit{iff} \;\; M \models \Box\,((S \cap S') \cup E \supset L)\,.$$

It follows that S' is a fixpoint of Equation (5) if and only if S' belongs to $Res^3_B(E, S)$.

Let ϕ be a formula of propositional logic. It is interesting to observe that in $\mathcal{C}_{\text{flat}}$ the state constraint $\Box\phi$ is logically equivalent to the s-conditional $\neg\phi \Rightarrow False$, which has the form of a qualification constraint. This may seem puzzling, since qualification constraints do not lead to ramifications, while state constraints traditionally have been used to do precisely that. The puzzle is resolved, however, by recalling that in the causal theory of Definition 4′ a requirement stronger than the usual one is placed on ramifications: namely, the requirement that ramifications be caused (as in Equation 4) and not merely implied (as in Equation 5).

[8]The elements of $\Box B$ are state constraints; each of them says of the corresponding formula in B that it is true in every possible state of the world.

Acknowledgements

The authors would like to thank Vladimir Lifschitz for many useful discussions on the subject of this paper. We are also grateful to Nicholas Asher, Chitta Baral, Enrico Giunchiglia, G. N. Kartha, and Rob Koons for their comments.

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