CS311: Discrete Math for Computer Science, Spring 2015

Homework Assignment 1, with Solutions

1. Simplify the given formulas.

(a) $n > 4 \wedge n < 6$. *Answer:* $n = 5$.

(b) $x > 3 \vee x < 3$. *Answer:* $x \neq 3$.

(c) $\neg(x > 10)$. *Answer:* $x \leq 10$.

2. Determine whether the given formula is true or false. If it is true then find a witness:

(a) $\exists x(2x^2 + 3x + 1 < 0)$.

Answer: True; witness: $x = -.6$. *Proof.* The polynomial $2x^2 + 3x + 1$ turns into 0 when $x = -1$ and when $x = -\frac{1}{2}$, and its value at any point between these two numbers is negative.

(b) $\exists xy(2x + y = 5 \wedge x + 2y = 6 \wedge x < y)$.

Answer: True; witness: $x = \frac{4}{3}$, $y = \frac{7}{3}$. *Proof.* These values of x and y form a solution to the equations $2x + y = 5$, $x + 2y = 6$, and they satisfy the additional condition $x < y$.

(c) $\exists mn(m^2 + n^2 = 6)$.

Answer: False. *Proof.* Number 6 cannot be represented as the sum of two complete squares, because the only complete squarest that do not exceed 6 are 0, 1, and 4.

3. Determine whether the given formula is true or false. If it is false then find a counterexample:

(a) $\forall n(2^n > 1 \vee n < 0)$. *Answer:* false; counterexample: $n = 0$.

(b) $\forall n \left(n^2 > 2^{-\frac{1}{2}}\right)$. *Answer:* false; counterexample: $n = 0$.

(c) $\forall xy(x^2 + y^2 = x^3 + y^3)$. *Answer:* false; counterexample: $x = y = 2$.

4. Translate into logical notation:

There exists a pair of negative integers such that their product is 6.

Find a witness showing that this assertion is true.

Answer: $\exists mn(m < 0 \wedge n < 0 \wedge mn = 6)$; witness: $m = -2$, $n = -3$.