

CS311: Discrete Math for Computer Science, Spring 2015

Homework Assignment 10, with Solutions

Determine whether the given formula is true or false. If true, prove it by turning it into a complete annotated program. If false, find a counterexample.

1. $\{i < 10 \wedge j < 10\} \ m \leftarrow i + j \ \{m < 10\}$.

Solution: False; counterexample: $i = j = 9$.

2. $\{i < 10 \wedge j < 10\} \ m \leftarrow i + j \ \{i + m < 100\}$.

Solution:

$$\begin{array}{l} \{i < 10 \wedge j < 10\} \\ \{2i + j < 30\} \\ \{2i + j < 100\} \\ \{i + i + j < 100\} \\ m \leftarrow i + j \\ \{i + m < 100\} \end{array}$$

3. $\{i = j\} \ i \leftarrow i + 1; \ j \leftarrow j + 1 \ \{i = j\}$.

Solution:

$$\begin{array}{l} \{i = j\} \\ \{i + 1 = j + 1\} \\ i \leftarrow i + 1; \\ \{i = j + 1\} \\ j \leftarrow j + 1 \\ \{i = j\} \end{array}$$

4. $\{i = j\} \ i \leftarrow i + 1; \ j \leftarrow j - 1 \ \{i - j > 1\}$.

Solution:

$$\begin{array}{l} \{i = j\} \\ \{i - j + 2 = 2\} \\ \{i - j + 2 > 1\} \\ \{(i + 1) - (j - 1) > 1\} \\ i \leftarrow i + 1; \\ \{i - (j - 1) > 1\} \\ j \leftarrow j - 1 \\ \{i - j > 1\} \end{array}$$

5. $\{true\} \ i \leftarrow j; \ j \leftarrow j + 1 \ \{i < j\}$.

Solution:

$$\begin{aligned} &\{true\} \\ &\{j < j + 1\} \\ &i \leftarrow j; \\ &\{i < j + 1\} \\ &j \leftarrow j + 1 \\ &\{i < j\} \end{aligned}$$

6. $\{true\} \ i \leftarrow j; \ j \leftarrow k \ \{i = k\}$.

Solution: False; counterexample: $i = j = 0, \ k = 1$.

- 7.

$$\begin{aligned} &\{n = 10\} \\ &\mathbf{if} \ m > n \\ &\quad \mathbf{then} \ p \leftarrow m \\ &\quad \mathbf{else} \ p \leftarrow n \\ &\mathbf{endif} \\ &\{p \geq 10\} \end{aligned}$$

Solution:

$$\begin{aligned} &\{n = 10\} \\ &\mathbf{if} \ m > n \\ &\quad \mathbf{then} \ \{n = 10 \wedge m > n\} \\ &\quad \{m > 10\} \\ &\quad \{m \geq 10\} \\ &\quad p \leftarrow m \\ &\quad \{p \geq 10\} \\ &\quad \mathbf{else} \ \{n = 10 \wedge m \leq n\} \\ &\quad \{n = 10\} \\ &\quad \{n \geq 10\} \\ &\quad p \leftarrow n \\ &\quad \{p \geq 10\} \\ &\mathbf{endif} \\ &\{p \geq 10\} \end{aligned}$$

- 8.

$$\begin{aligned} &\{m = n + 1\} \\ &\mathbf{if} \ m > n \\ &\quad \mathbf{then} \ p \leftarrow m \\ &\quad \mathbf{else} \ p \leftarrow n \\ &\mathbf{endif} \\ &\{p = m\} \end{aligned}$$

Solution:

```
{ $m = n + 1$ }  
if  $m > n$   
  then { $m = n + 1 \wedge m > n$ }  
    { $m = m$ }  
     $p \leftarrow m$   
    { $p = m$ }  
  else { $m = n + 1 \wedge m \leq n$ }  
    {false}  
    { $n = m$ }  
     $p \leftarrow n$   
    { $p = m$ }  
endif  
{ $p = m$ }
```