

Written in anger.

"But the pictures are not the subject matter of geometry and we are not permitted to reason from them. It is true that most people including mathematicians, lean upon these pictures as a crutch and find themselves unable to walk when the crutch is removed."

Morris Kline in the chapter "A Discourse on Method" from "Mathematics in Western Culture", Oxford University Press, Inc., 1953

Morris Kline's observation is correct; he omitted the explanation of what he had observed, although the explanation is simple: most people, including mathematicians, are amateur thinkers in the sense that they have not been taught how to think effectively. They have not been told to throw the crutch away and, as a result, have never learned how to run.

The habit of using pictorial aids, like any habit, is very difficult to get rid of. If, however, we take any responsibility for the effectiveness of our thinking habits, we should try to get rid of the habit as quickly as possible, for it is a bad habit, confusing and misleading up to the point of being paralysing. One of the bad things about pictures is that they are almost always overspecific. One cannot make a picture of "an arbitrary triangle": as soon as one has made it, it has either an obtuse angle or not, whereas for "the arbitrary triangle" the property of having an obtuse angle is explicitly undefined. In the case of graphs it is even worse, because the same specific graph has so many pictorial representations that just to establish that two different pictures represent the same graph may require a clumsy checking process. In the case of trees and lists a nicely confusing circumstance is that most pictorial conventions do not include a visible representation for the empty tree, nor for the empty list. They are misleading because the same thing has many visibly very different pictures; their use is confusing because it is very rare, when an author states when two different pictures are to be regarded as semantically equivalent, and they are paralysing because they

can only represent individual members of a set. And when dealing with a set, one of the worst possible mistakes one can make while thinking is trying to come to grips with the set as a whole by dealing with the individual members of a subset of which one can only pray that it is representative: one can only deal with a set --and thereby with all its members-- via its definition. Once you have grasped this, it is not amazing to hear that a major component of learning how to think effectively, is the "unlearning" of the use of pictures. (And "unlearning" is very difficult, as your past remains your past: the only thing you can do is to superimpose a new past on top of the old one, and pray that the more recent past will be dominant.)

And all this has been triggered by "Design of Abstract Programs in an Interactive Environment", the doctoral thesis of Lars Kahn from Stockholm, who kindly sent me a copy, which I studied the other night. Among the many remarks I shall not quote, (now Dr.) Lars Kahn states "[that it] is my own and other's experience that it is more natural in the design process to use graphical notation than text. I have for various reasons not had the opportunity to implement an interactive tool with graphic notation, but I believe that an easily managed graphic display system suited for program design would be the best mental aid." Here "more natural" should be read as "more natural for the uneducated": his "easily managed graphic display system" would be the most severe disservice to program design that I can imagine. The Dissertation --but that I only saw afterwards, having skipped the small print-- was for the Doctor's Degree in the social sciences, which are certainly about the uneducated. Sometimes I fear that they are also for the uneducated, and by the uneducated.

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Plataanstraat 5
5671 AL Nuenen
The Netherlands

prof.dr.Edsger W.Dijkstra
Burroughs Research Fellow