A postscript to EWD755.

While presenting my proof of Theorem 2 in EWD755 I had forgotten my own preaching (in EWD731): with explicitly named predicates a more symmetric presentation is possible. With

C n p = n is the product of a bag of primes containing p
D n p = n is the product of a bag of primes not containing p

we have (as obviously as before) for positive integers p, x, and y :

1) $\neg(\text{prime } p) \vee \neg(p \mid (x.y)) \vee (C\,(x.y)\;p)$
2) $p \mid x \vee p \mid y \vee (D\,(x.y)\;p)$
3) $\neg(C\,(x.y)\;p) \vee \neg(D\,(x.y)\;p) \vee \neg(UPF(x.y))$

Theorem 2 now follows by the standard inference rule. Note that for 1) and 2) we need Theorem 0. (In EWD755 I failed to mention this.)

* * *

The above I regard as a different (and preferable) presentation of the same proof as in EWD755. The following alternative proof for Theorem 0 from EWD755 I consider (perhaps somewhat arbitrarily) different.

Consider, for $n \geq 1$, the following program

if $n = 1 \rightarrow bag := \emptyset$ [] $n > 1 \rightarrow bag := \{n\}$ **fi**;
do bag contains a composite multiple →
    replace each occurrence of the
    largest composite multiple c in bag
    by the multiples x and y, where
    $x \cdot y = c$
od

The repetition leaves the product of the numbers in bag equal to n. On account of (3') from EWD755 - which is, of course, again needed - the lowest upper bound for composite multiples in bag can be taken as variant function, and termination is guaranteed. This proof is more constructive; Euclid would have liked it (if he did not prove it that way!).

Plataanstraat 5
5671 AL NUENEN
The Netherlands

23 October 1980
prof. dr. Edsger W. Dijkstra
Burroughs Research Fellow