Exercise 5 – Finite Element Meshing - I: Linear Elements

CS384R, CAM 395T, BME 385J: Fall 2007

Out: November 2, Due: November 12

Question 1. One measure of a quality tetrahedron T is the aspect ratio bound, γ, where γ = ratio of the radii of the circumscribing sphere of T and the inscribed sphere of T . What is γ for a regular tetrahedron ?

Question 2. For a tetrahedron T, consider the mid-edge decomposition of T, which splits T into four-sub tetrahedra. If T is initially a regular tetrahedron, what is γ for the four-sub tetrahedra of T under mid-edge subdivision?

Question 3. For a tetrahedron T, consider choosing a point p inside of T, which if joined to the four vertices of T yields a 4-way split of T into sub-tetrahedra. Which of the following choices of p yields the best γ split of T : (a) p is the center of the circumscribing sphere of ? (b) p is the center of the inscribed sphere of T ? (iii) p is the centroid of T?

Question 4. Describe two ways to decompose a cube into tetrahedra, without using any Steiner points (i.e. no additional vertices other than the original vertices). Which decomposition yields better γ for the resulting tetrahedra ?

Question 5. How many ways are there to decompose an octahedron, an icosahedron, and a dodechadron into tetrahedra without Steiner points. ?

Question 6. One measure of a quality quad element Q or hex element H (also called a hexahedron or brick element) is the Jacobian norm J. Assume $x \in \Re^3$ is a vertex of the quad or a hex, and $x_i \in \Re^3$ for $i = 1, \ldots, m$ are its neighboring vertices, where $m = 2$ for a quad and $m = 3$ for a hex. Edge vectors are defined as $e_i = x_i - x$ with $i = 1, \ldots, m$, and the Jacobian norm is $det([e_1, ..., e_m])$. (a) What is J for the unit square and the unit cube ? (b)When is J zero for a quad or a hex ? (c) When is J negative for a quad or hex ?

Question 7. Given a collection of n disjoint triangles T_i of different sizes within a bounding rectangle D, describe a method to decompose the region bounded by (D - union of all T_i) into quadrilaterals of nice quality i.e. decompose the region inside D but outside each triangle T_i into quads, all with positive Jacobian norms J.