Exercise 6: Finite Element Meshing II - Non-Linear Elements

CS384R, CAM 395T, BME 385J: Fall 2007

Due: Nov 26^{th}

1. Given a collection of circles (not necessarily disjoint) in a rectangular domain D, describe a method to partition the domain D into a quad mesh such that each quad contains at most a single circular arc through it.

2. For a quadrilateral with a single circular arc, give the lowest degree polynomial or rational bivariate function basis so that the A-spline (zero set of the function on the quad) recovers the circular arc exactly.

3. Consider a bilinear mapping $BL : R^2 \rightarrow R^2$ from (UV) space to (XY). Derive the implicit equation in X, Y of the image under the BL mapping of lines in the unit U, V domain (Fig 1). Express the equation as an A-spline over the (X, Y) region spanned by $\left[\overrightarrow{p_0} \quad \overrightarrow{p_1} \quad \overrightarrow{p_2} \quad \overrightarrow{p_3} \right]$.

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} (1-U)(1-V) & U(1-V) \end{bmatrix} \begin{bmatrix} \overrightarrow{p_0} \\ \overrightarrow{p_1} \end{bmatrix} + \begin{bmatrix} (1-U)V & UV \end{bmatrix} \begin{bmatrix} \overrightarrow{p_2} \\ \overrightarrow{p_3} \end{bmatrix}$$

4. Consider the following mapping from a unit triangular straight prism in UVS space into an irregular prism in XYZ space (Fig 2) :

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} (1-U-V)(1-S) & U(1-S) & V(1-S) \end{bmatrix} \begin{bmatrix} \overrightarrow{p_1} \\ \overrightarrow{p_2} \\ \overrightarrow{p_3} \end{bmatrix} + \begin{bmatrix} (1-U-V)S & US & VS \end{bmatrix} \begin{bmatrix} \overrightarrow{p_4} \\ \overrightarrow{p_5} \\ \overrightarrow{p_6} \end{bmatrix}$$

   Let the family of planes P , Q and R be respectively the $U =$ constant, $V=$ constant and $S =$ constant family of planes in the triangular prism. Derive the implicit equations in (X, Y, Z) of the image under the mapping of each of these family of planes P, Q, and R . Express these implicit equations as A-patches over the irregular prism.

Note: The problems in this exercise, indicate of an effective methodology to generate non-linear finite elements, via partition of unity mappings applied to linear or bi-linear finite elements. These non-linear finite elements have both an implicit and parametric representation.

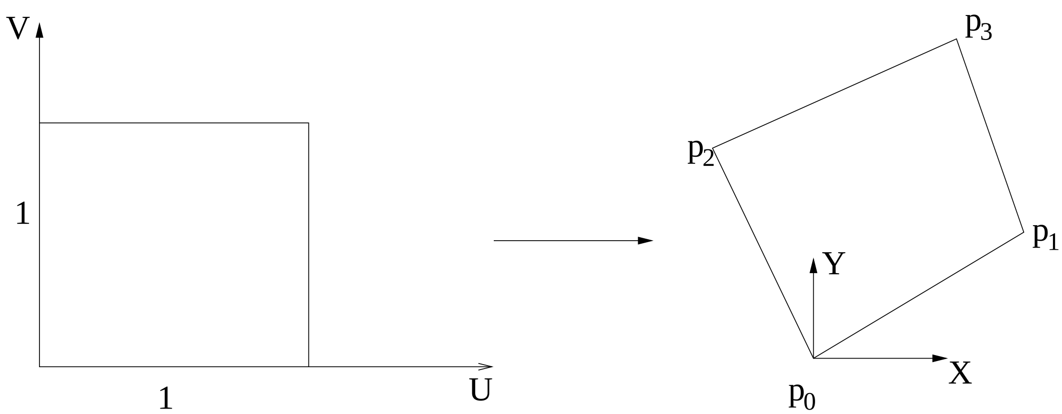

Figure 1: Problem 3

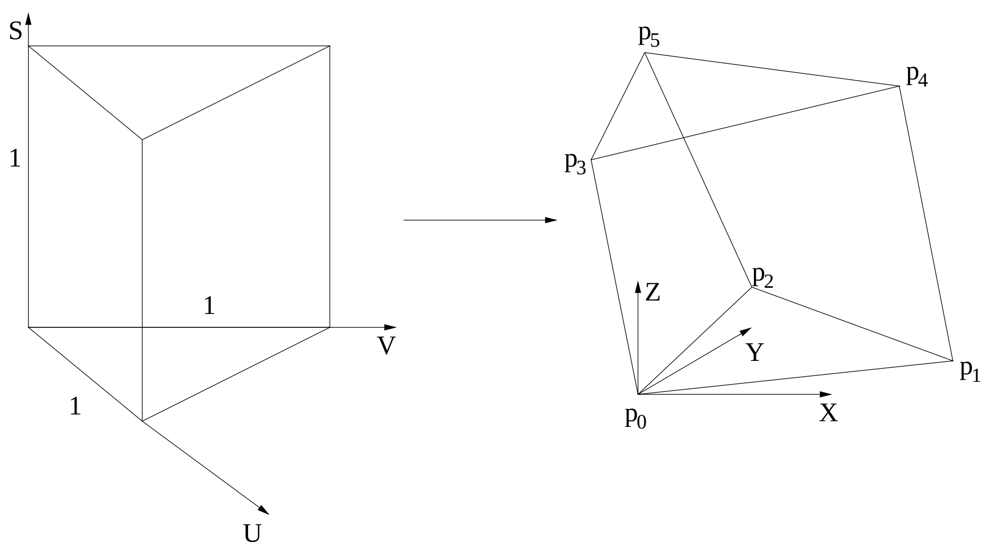

Figure 2: Problem 4