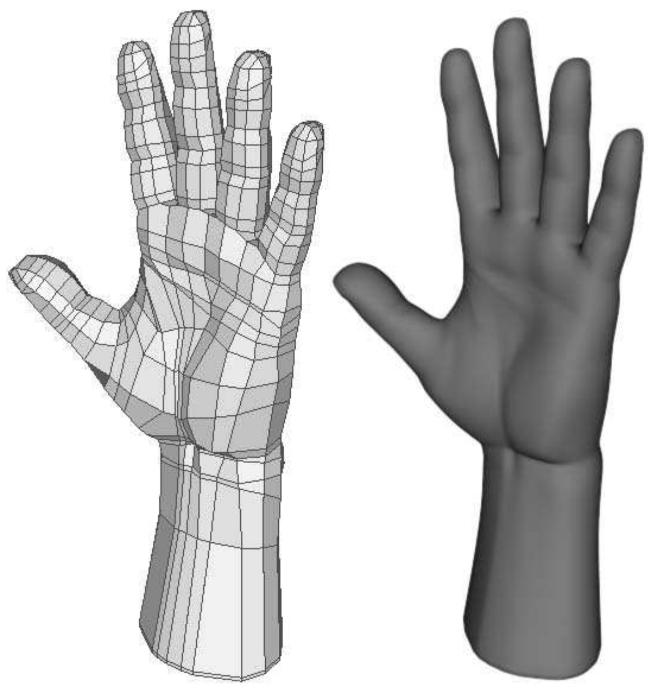
Curves, Surfaces and Recursive Subdivision

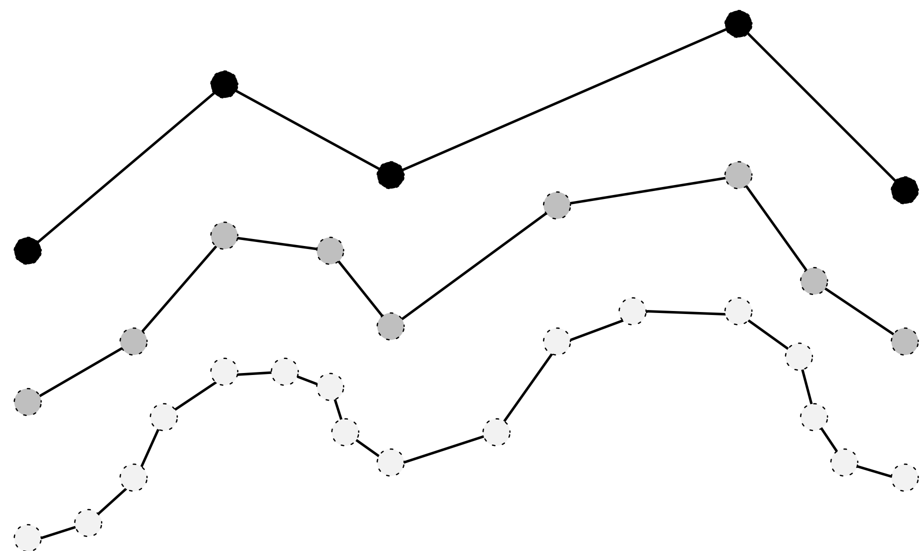
- Recursive Subdivision of Curves, Polygons
- Recursive Subdivision of Polyhedra



B-Splines via subdivision

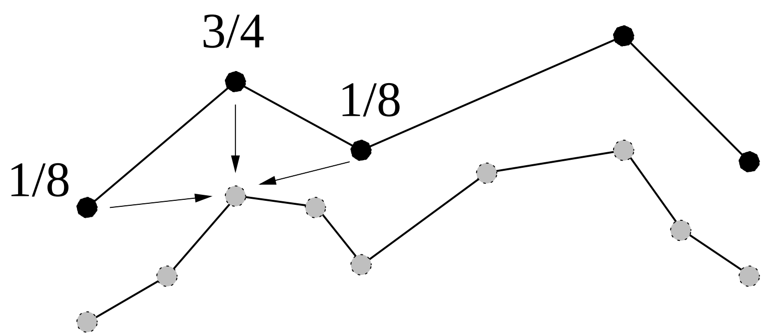
It turns out that the smooth curve be obtained by subdivision of the original polyline:

Subdivision adds new control points between the original control points and updates positions of original control points.

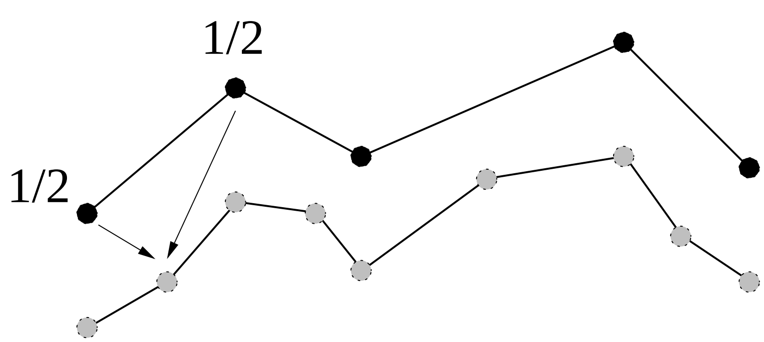


Subdivision

Subdivision rules for updating old points:



Subdivion rules for inserting new points:



Subdivision rules

Even rule (in the new sequence of points the points with even numbers are the old points with updated positions).

$$p_{2i}^{j+1} = \frac{1}{8}(p_{i-1}^{j} + 6p_{i}^{j} + p_{i+1}^{j})$$

Odd rule (in the new sequence the points with odd numbers are newly inserted points).

$$p_{2i+1}^{j+1} = \frac{1}{8}(4p_{i}^{j} + 4p_{i+1}^{j})$$

Subdivision

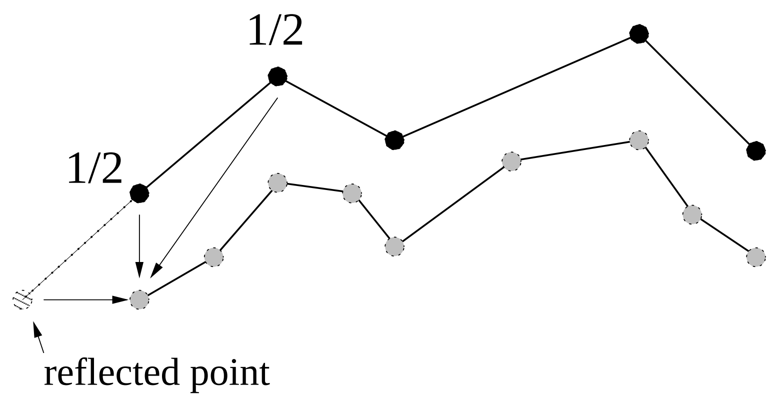
Of course, in a finite number of steps subdivision generates only polylines. But they get arbitrarily close to a limit C^2 and this curve is exactly a cubic B-spline.

Algorithm:

Start with an array of control points of length $n + 1$.
Compute from the original points new array of length $2n + 1$ using subdivision rules for even and odd points.
Then from the new array compute an array of length $4n + 1$ etc., (typically 4-5 steps is enough).
Then draw the line segments connecting sequential control points.

Endpoints

What do we do when a point is missing?



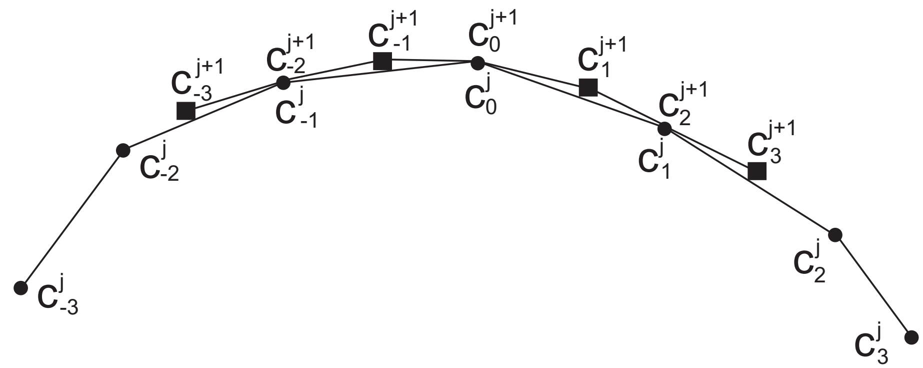
In this case, apply reflection (recall adding missing control points). This results in the following trivial rule:

$$p_0^{j+1} = \frac{1}{8}(p_1^j + 6p_0^j + (2p_0^j - p_1^j)) = p_0^j$$

that is, just keep the old value.

Subdivision of Polygons

Four Point Scheme



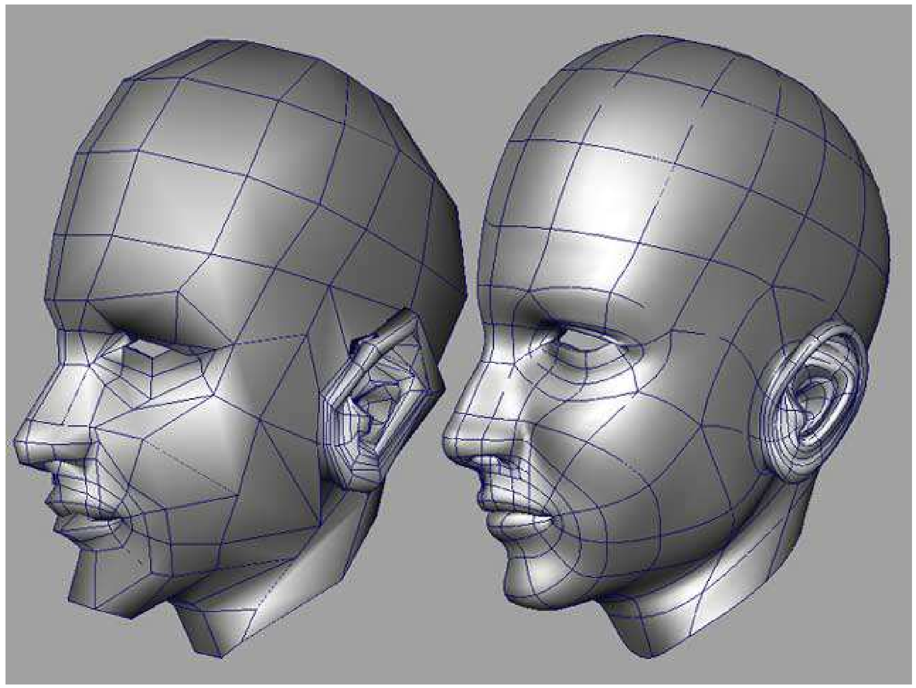
Four point scheme: the filled circles are the level j control points, the filled squares are the level $j+1$ control points.

For four-point scheme we need to consider only 7 control points; these 7 points completely define the piece of the curve around a control point. We can consider a set of 7 control points *on any subdivision level*, as we do not care how small our piece of the curve is. Note that we can compute the positions of the seven control points on level $j+1$ from the positions of similar seven control points on level j, using a 7×7 submatrix $\mathbf{S}$ of the infinite subdivision matrix.

The local subdivision matrix for the four-point scheme is:

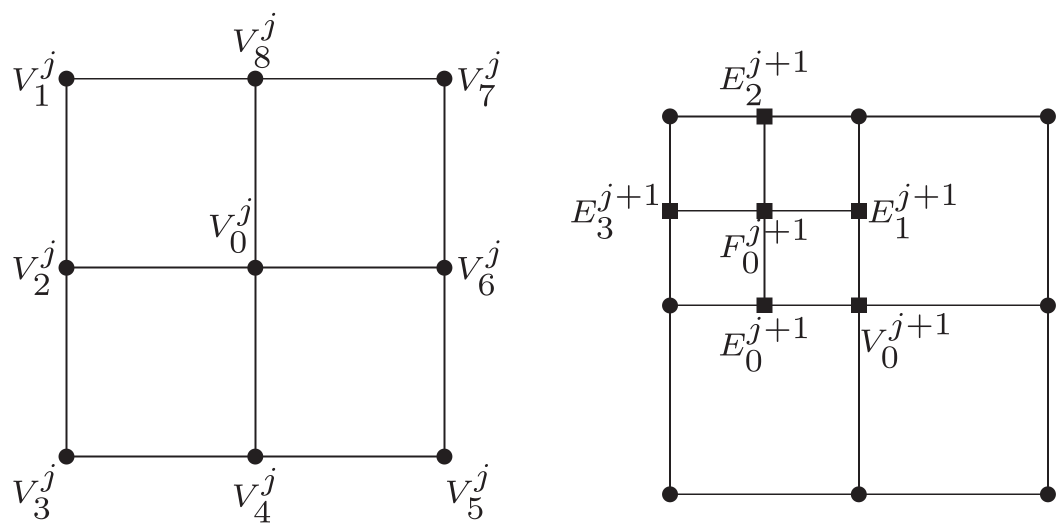
$$\begin{pmatrix} c_{-3}^{j+1} \\ c_{-2}^{j+1} \\ c_{-1}^{j+1} \\ c_{0}^{j+1} \\ c_{1}^{j+1} \\ c_{2}^{j+1} \\ c_{3}^{j+1} \end{pmatrix} = \begin{pmatrix} -\frac{1}{16} & \frac{9}{16} & \frac{9}{16} & -\frac{1}{16} & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{16} & \frac{9}{16} & \frac{9}{16} & -\frac{1}{16} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{16} & \frac{9}{16} & \frac{9}{16} & -\frac{1}{16} & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{16} & \frac{9}{16} & \frac{9}{16} & -\frac{1}{16} \end{pmatrix} \begin{pmatrix} c_{-3}^{j} \\ c_{-2}^{j} \\ c_{-1}^{j} \\ c_{0}^{j} \\ c_{1}^{j} \\ c_{2}^{j} \\ c_{3}^{j} \end{pmatrix}$$

Recursive Subdivision of Surfaces



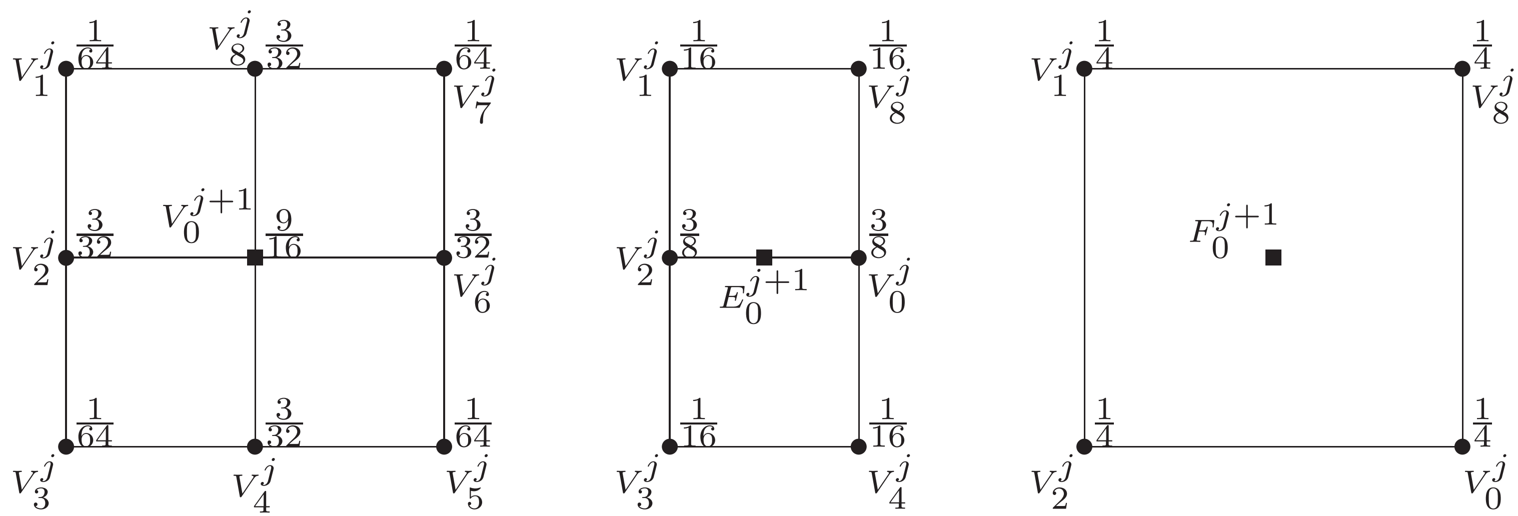
Catmull Clark

Refinement rule used by Catmull-Clark subdivision scheme is as follows. New vertices are added on each edge and in the center. When connected, 4 new level $j+1$ quadrilaterals are produced from the single level j quadrilateral.



Catmull-Clark subdivision scheme. Circles are the j level and Squares are the $j+1$ level.

The vertex rule, edge rule and face rule are shown in the following figure. Each black circle represents a vertex at level j; we compute the position of the vertex at level $j+1$ marked by the black square. Note that for the vertex rule, the control vertex with weight $\frac{9}{16}$ and the new vertex aren't necessarily aligned as they are in the figure.



- Vertex rule:

$$V_0^{j+1} = \frac{9}{16}V_0^j + \frac{3}{32}(V_2^j + V_4^j + V_6^j + V_8^j) + \frac{1}{64}(V_1^j + V_3^j + V_5^j + V_7^j)$$

- Edge rule:

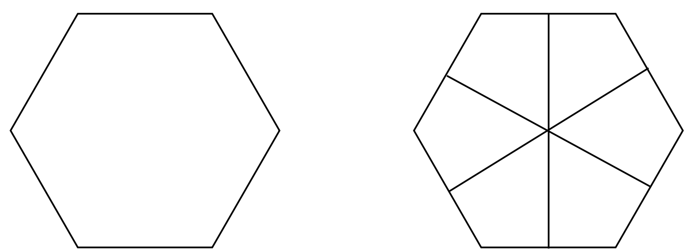
$$E_1^{j+1} = \frac{3}{8}(V_0^j + V_2^j) + \frac{1}{16}(V_1^j + V_3^j + V_4^j + V_8^j)$$

- Face rule:

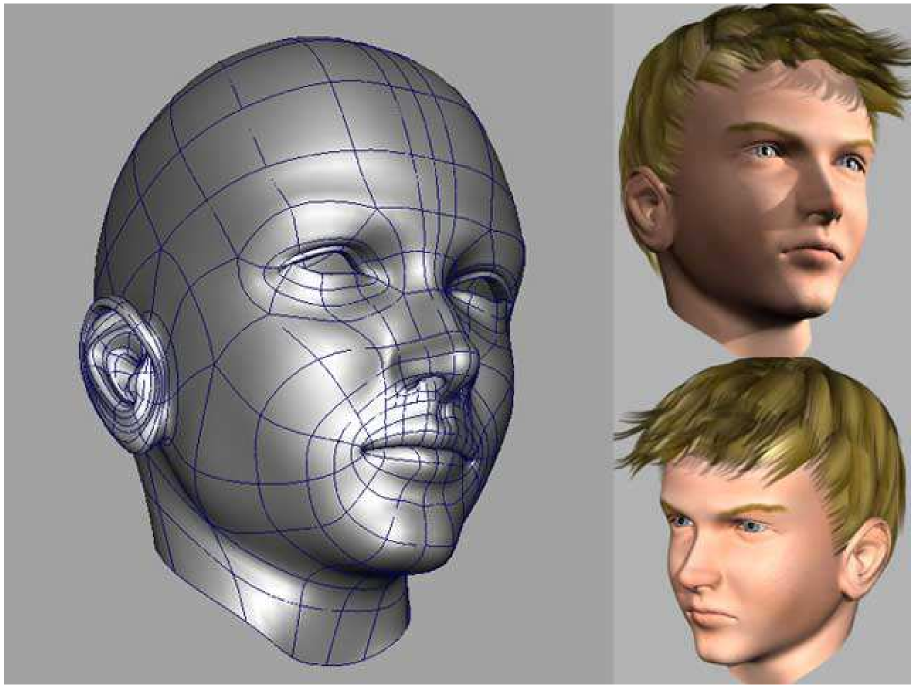
$$F_0^{j+1} = \frac{1}{4}(V_1^j + V_2^j + V_0^j + V_8^j)$$

Arbitrary Meshes

We have defined Catmull-Clark scheme on quadrilaterals; it can be extended to handle arbitrary polygonal meshes. Observe that if we do one step of refinement, splitting each edge into two and inserting a new vertex for each face (see below Figure), we get a mesh which has only quadrilateral faces. On all other steps of subdivision standard rule described above can be applied.



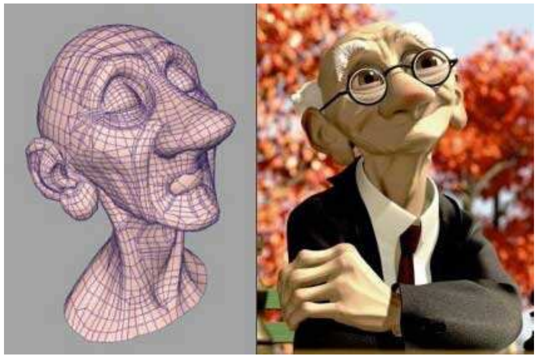
Splitting a hexagon into quadrilaterals.



Shapes and Scenes

Geometric Modeling Techniques for Shapes

- Non-Smooth Surfaces (Fractals, Polygon Soup)
- Interactive and Editable free-form surfaces (Subdivision Splines, A-splines, NURBS)
- Shell surfaces
- Boolean Set (CSG) operations on Solids
- Physically Based Procedural Modeling (Diffusion Modeling, Particle systems, Elasto-dynamics),



Scenes

Games, Movies, Advertisements, Scientific Discovery,

- Natural and Artificial Terrains
- Simulated Environments
- Nano-worlds to Cosmo- Worlds

Reading Assignment and News

We shall have Midterm 1 in the next class. Its open book, open notes, but no communication with anyone, the internet of Siri! Please review all lectures 0 - 12, (including supplementary) as well as the review notes h and practice exercises, and chapters 1 -4 and 10 of the recommended text.

(Recommended Text: Interactive Computer Graphics, by Edward Angel, Dave Shreiner, 6th edition, Addison-Wesley)

Please track Blackboard for the most recent Announcements and Project postings related to this course.

(http://www.cs.utexas.edu/users/bajaj/graphics2012/cs354/)