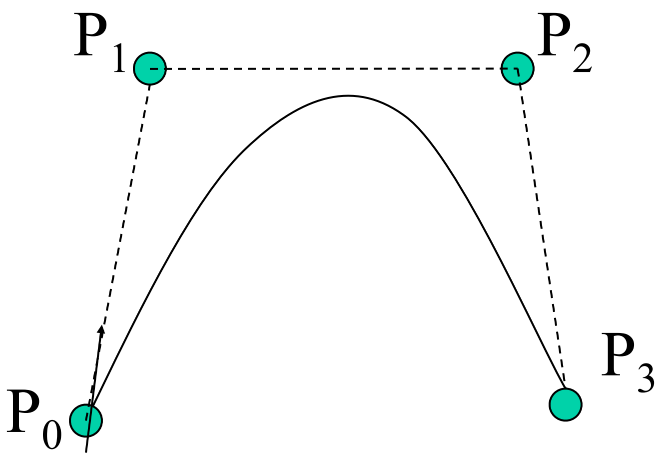
Bezier vs B-Spline

(Supplement Notes)

Bezier curve

- Developed by Paul de Casteljau (1959) and independently by Pierre Bezier (1962).
- French automobil company – Citroen & Renault.



Parametric function

- $P(u) = \sum_{i=0}^{n} B_{n,i}(u)p_i$

Where

$$B_{n,i}(u) = \frac{n!}{i!(n-i)!} u^i(1-u)^{n-i} \qquad 0 <= u <= 1$$

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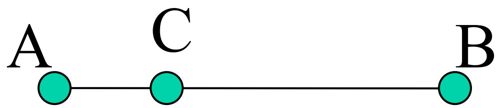
For four control points, n = 3

$P(u) = (1-u)^3p_0 + 3u(1-u)^2\, p_1 + 3u^2\,(1-u)p_2 + u^3p_3$

algorithm

- De Casteljau
  - Basic concept



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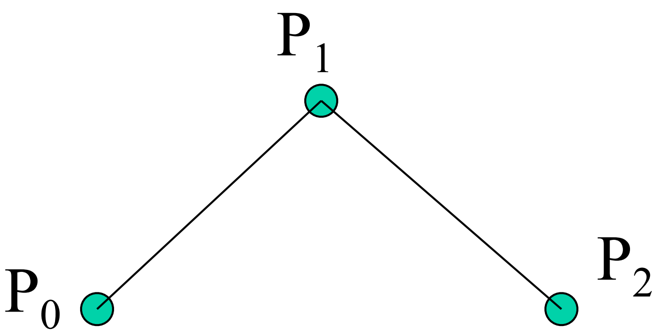

Let u = 0.5

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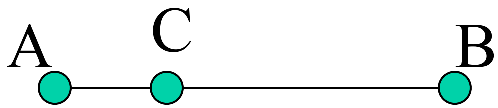

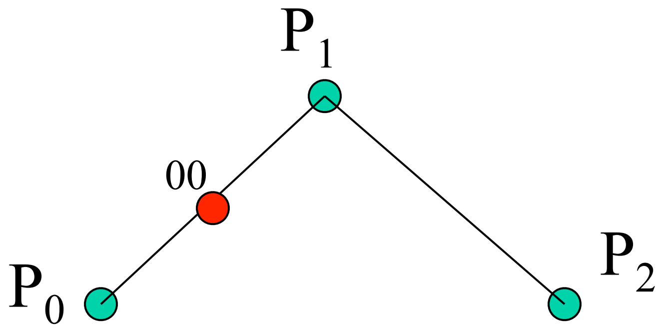

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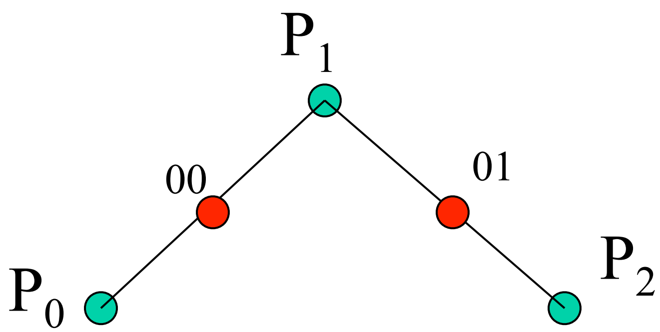

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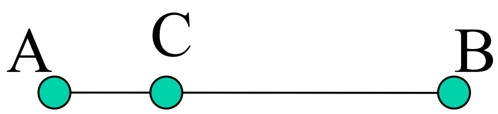

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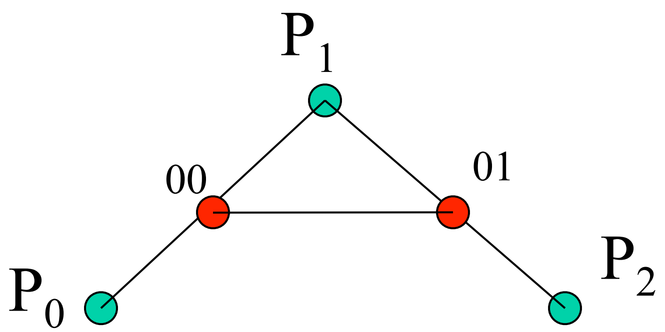

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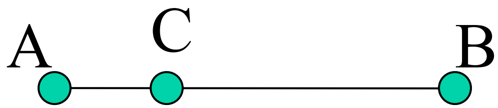

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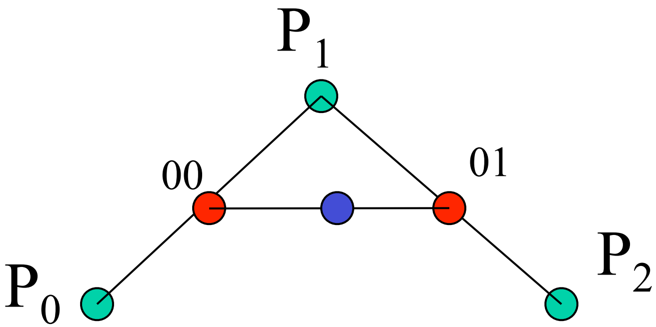

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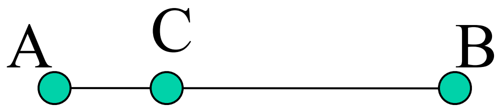

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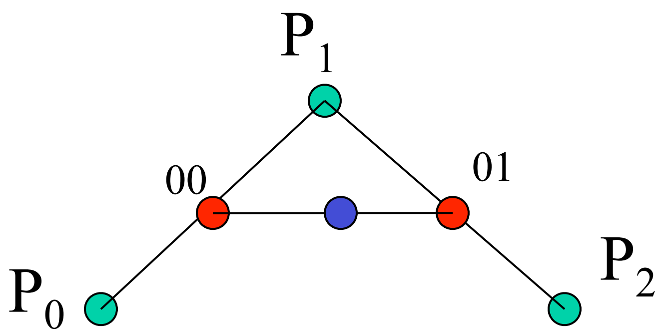

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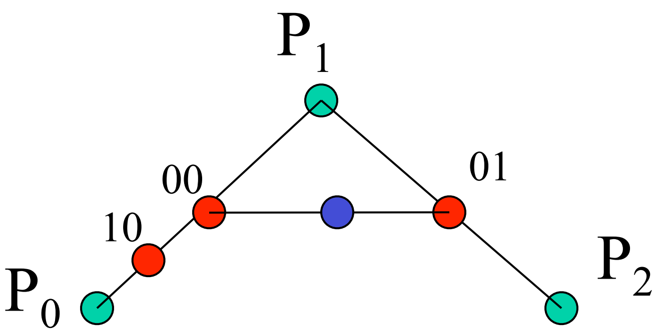

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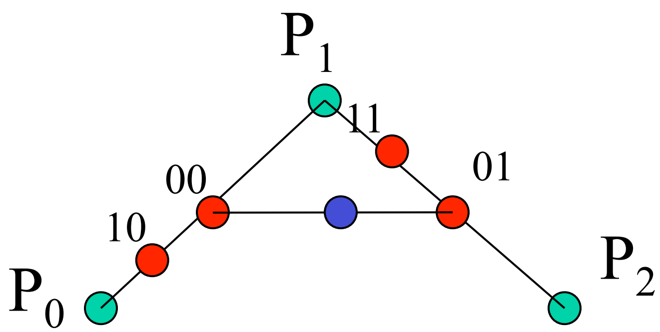

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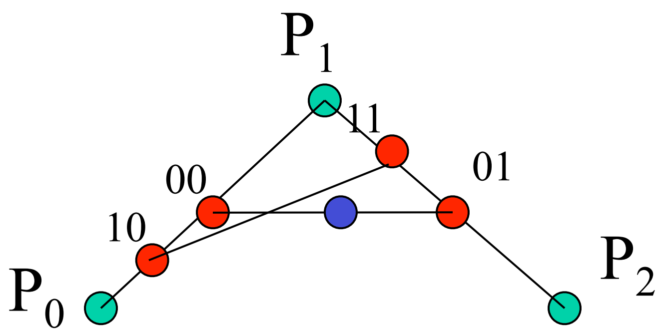

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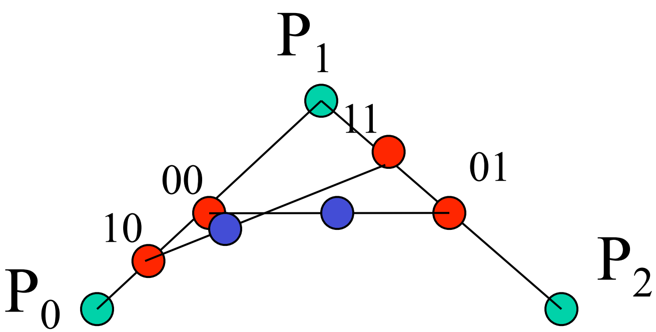

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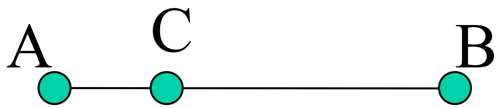

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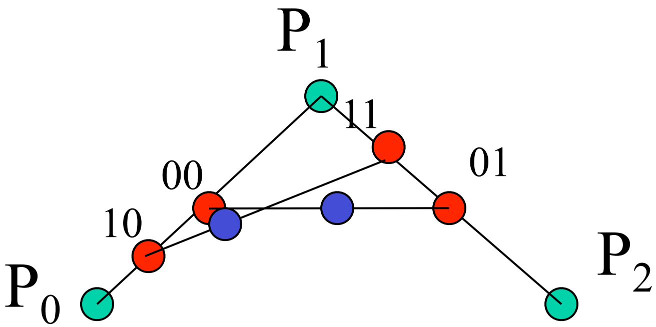

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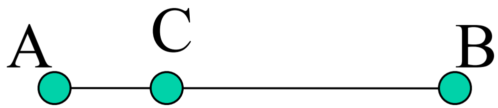

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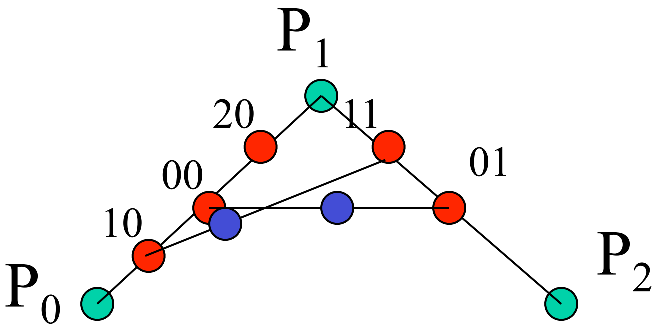

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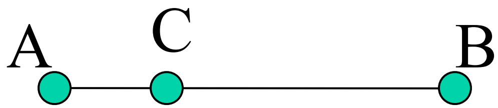

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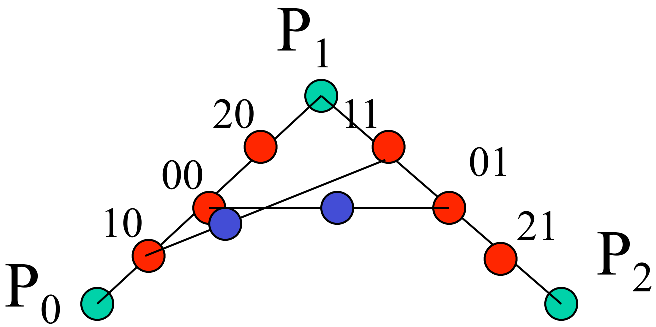

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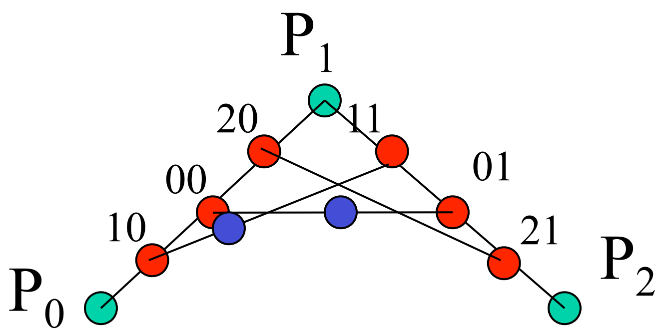

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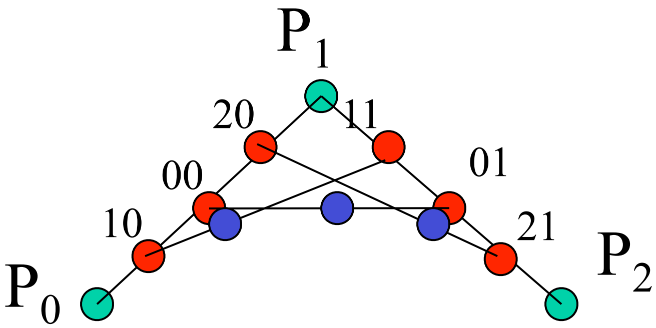

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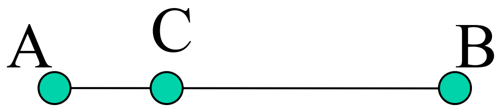

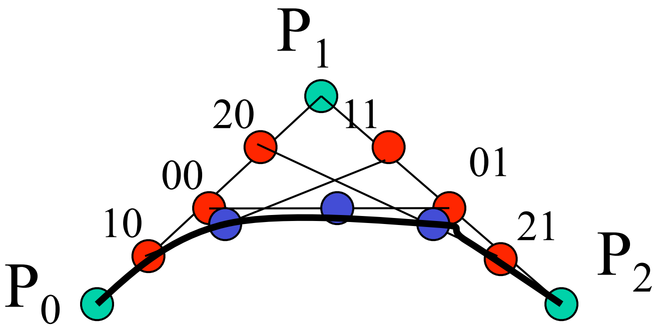

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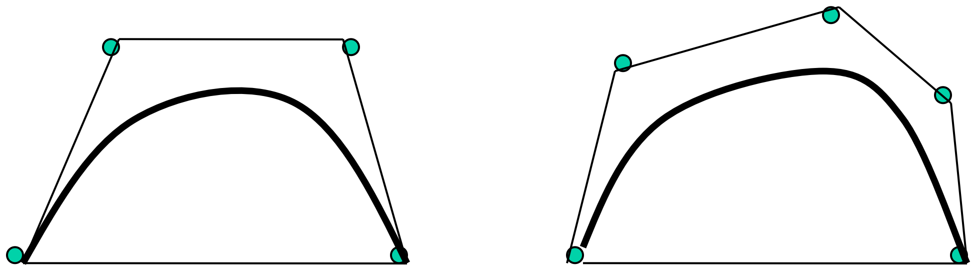
- The same curve is generated when the order of the control points is reversed

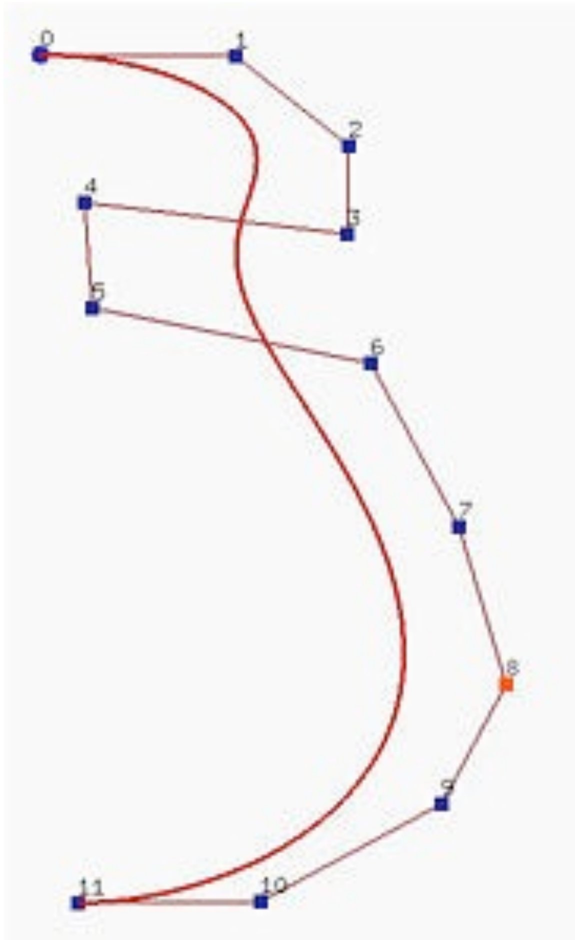
Properties (continued)

- Convex hull
  - Convex polygon formed by connecting the control points of the curve.
  - Curve resides completely inside its convex hull



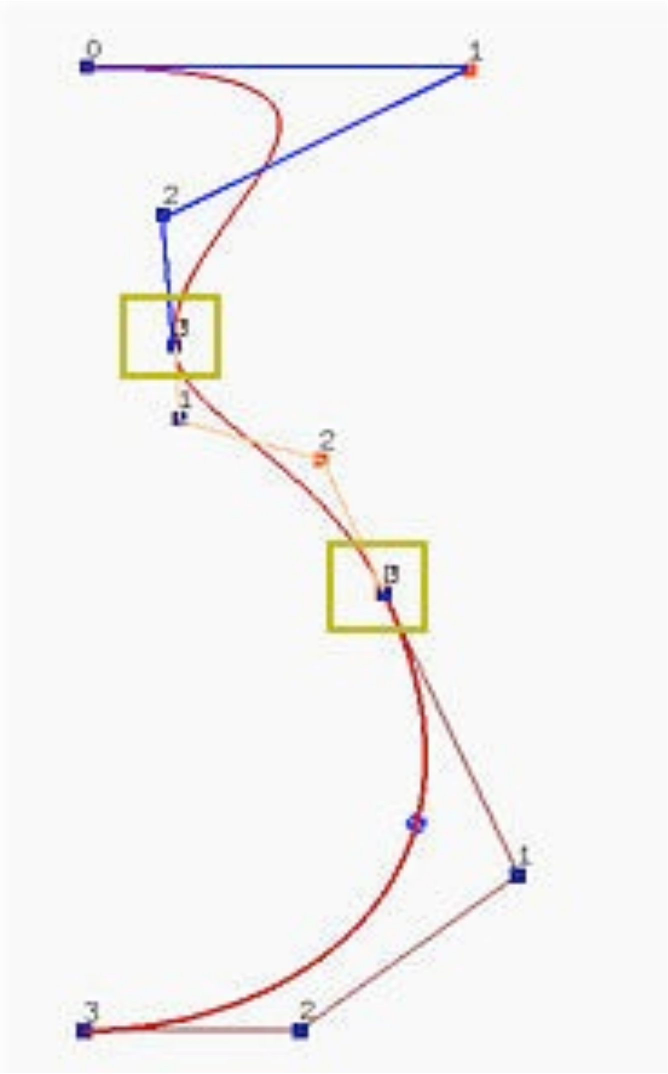
B-Spline

- Motivation (recall bezier curve)
  - The degree of a Bezier Curve is determined by the number of control points
  - E. g. (bezier curve degree 11) – difficult to bend the "neck" toward the line segment $\mathbf{P}_4\mathbf{P}_5$.
  - Of course, we can add more control points.
  - BUT this will increase the degree of the curve → increase computational burden



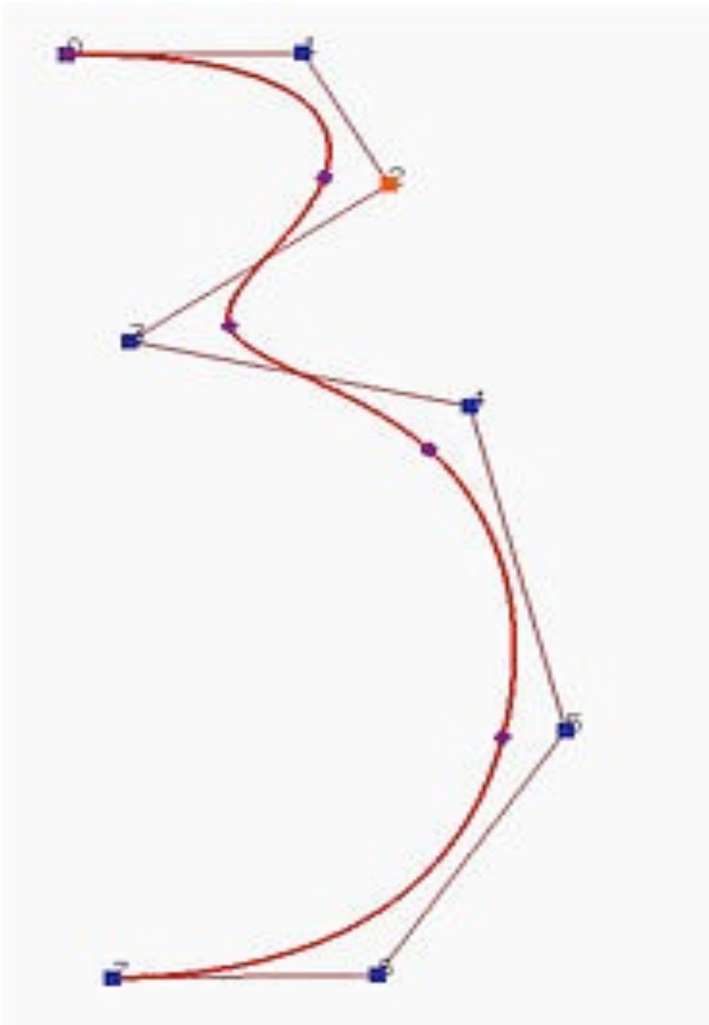
B-Spline

- Motivation (recall bezier curve)
  - Joint many bezier curves of lower degree together (right figure)
  - BUT maintaining continuity in the derivatives of the desired order at the connection point is not easy or may be tedious and undesirable.



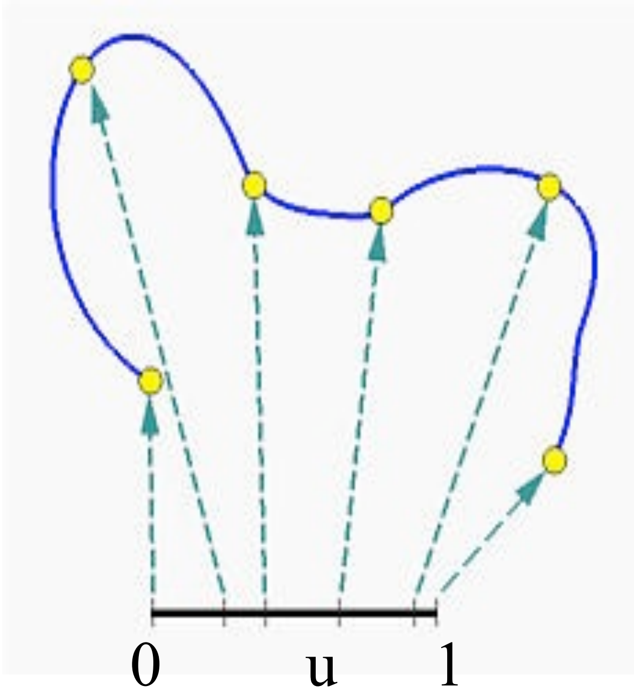
B-Spline

- Motivation (recall bezier curve)
  – moving a control point affects the shape of the entire curve- (*global modification property*) – undesirable.
  - Thus, the solution is B-Spline – the degree of the curve is independent of the number of control points
  - E.g - right figure – a B-spline curve of degree 3 defined by 8 control points



B-Spline

- In fact, there are five Bézier curve segments of degree 3 joining together to form the B-spline curve defined by the control points
- little dots subdivide the B-spline curve into Bézier curve segments.
- Subdividing the curve directly is difficult to do → so, subdivide the domain of the curve by points called *knots*



B-Spline

- In summary, to design a B-spline curve, we need a set of control points, a set of knots and a degree of curve.

B-Spline curve

- $P(u) = \sum_{i=0}^{n} N_{i,k}(u)p_i \qquad (u_0 < u < u_m)..$ (1.0)

Where basis function = $N_{i,k}(u)$

Degree of curve → k-1

Control points, p_i → $0 < i < n$

Knot, u → $u_0 < u < u_m$

$m = n + k$

B-Spline : definition

- $P(u) = \sum N_{i,k}(u)p_i$ $(u_0 < u < u_m)$
- u_i → knot
- $[u_i, u_{i+1})$ → knot span
- $(u_0, u_1, u_2, \ldots. u_m)$ → knot vector
- The point on the curve that corresponds to a knot u_i, → knot point ,$P(u_i)$
- If knots are equally space → uniform (e.g, 0, 0.2, 0.4, 0.6…)
- Otherwise → non uniform (e.g: 0, 0.1, 0.3, 0.4, 0.8 …)

B-Spline : definition

- Uniform knot vector
  - Individual knot value is **evenly spaced**
  - (0, 1, 2, 3, 4)
  - Then, normalized to the range [0, 1]
  - (0, 0.25, 0.5, 0.75, 1.0)

Type of B-Spline uniform knot vector

Non-periodic knots
(open knots)

Periodic knots
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-Curve doesn't pass through end points.
- used to generate closed curves (when first = last control points)

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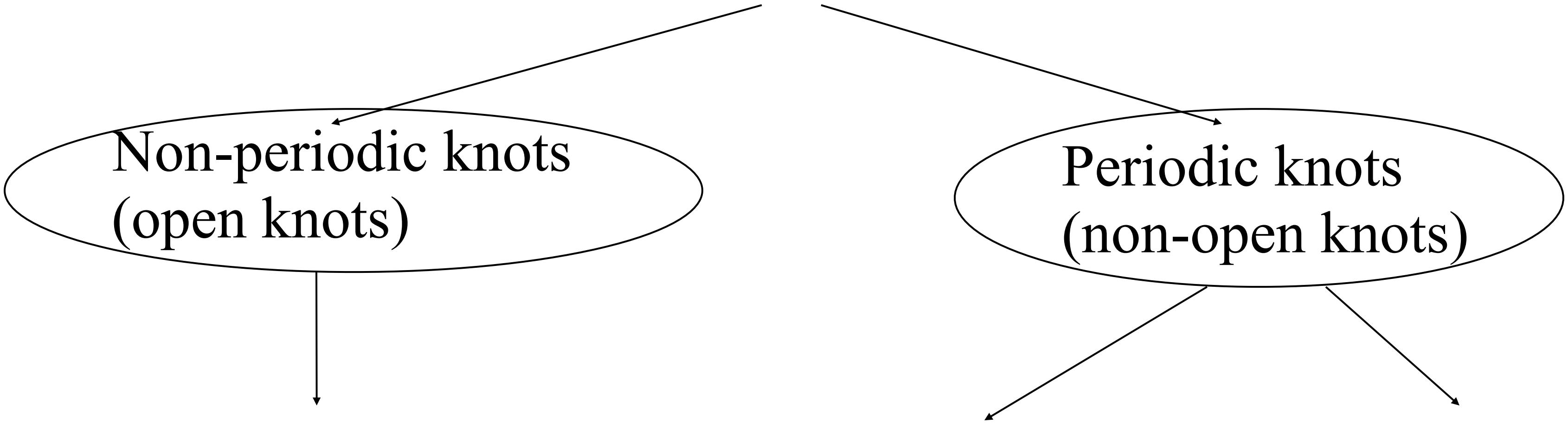

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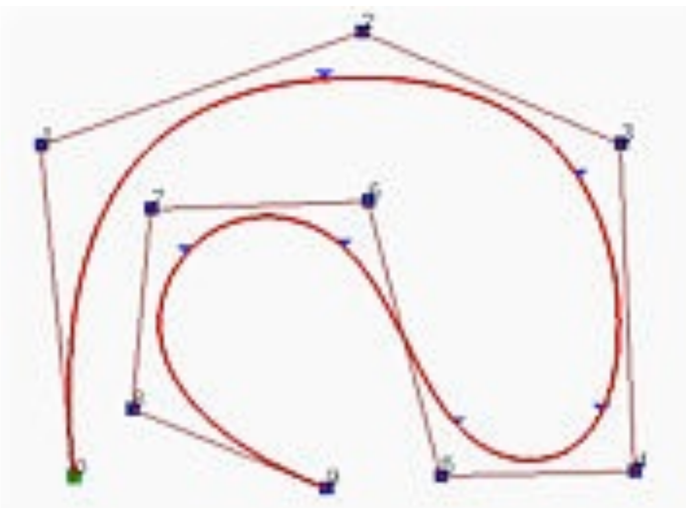

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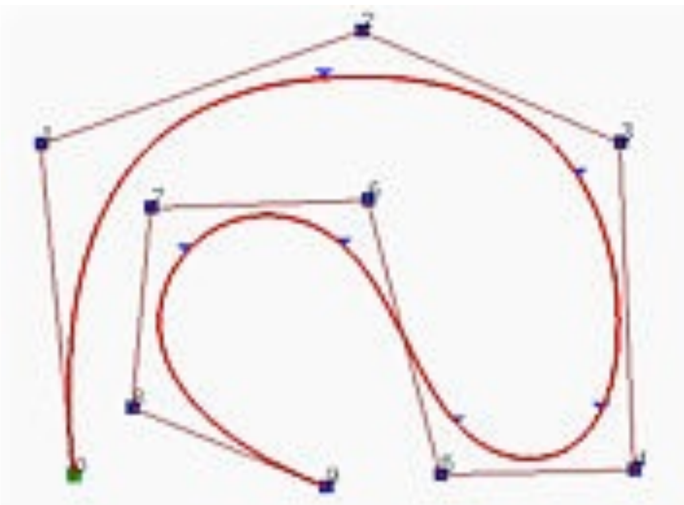

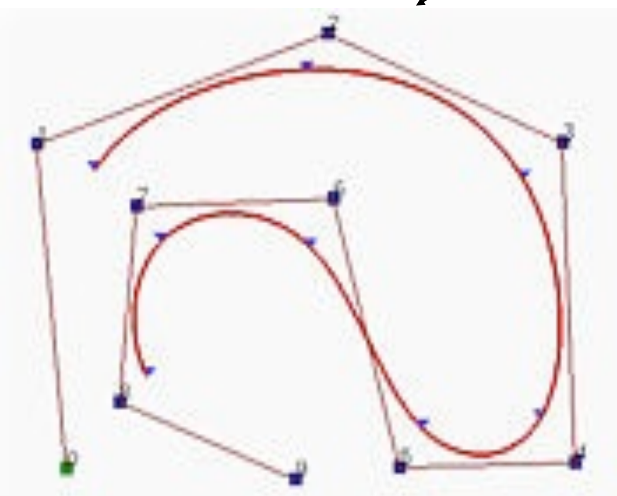

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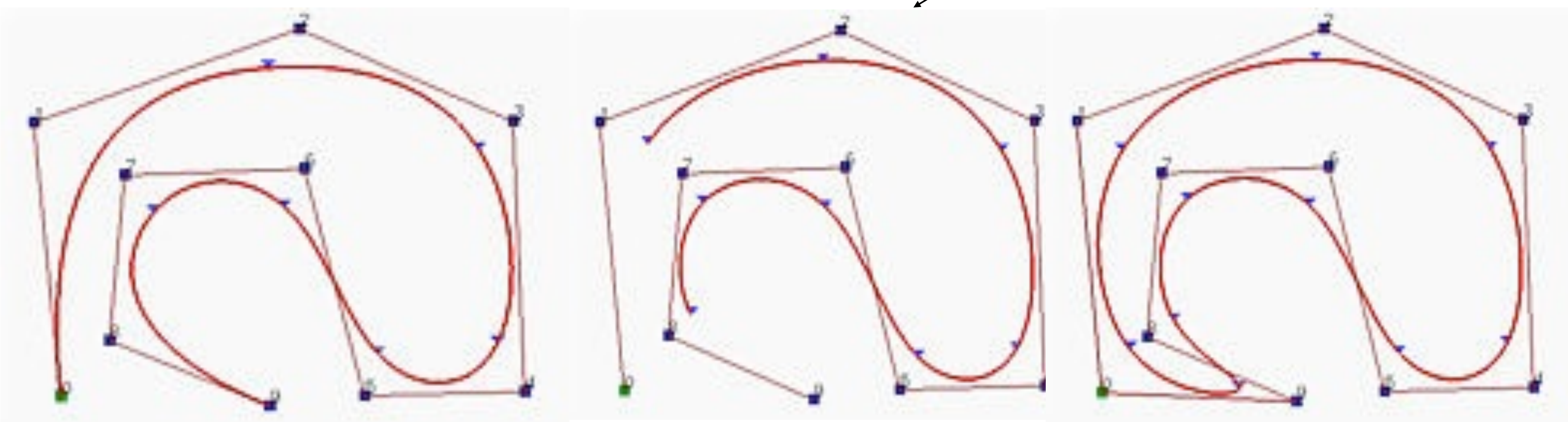

(Closed knots)

Non-periodic (open) uniform B-Spline

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i=0

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non periodic uniform knot vector (0,0,1,2,3, 3)

Non-periodic (open) uniform B-Spline

- The knot spacing is evenly spaced except at the ends where knot values are repeated *k* times.
- E.g $P(u) = \sum_{i=0}^{n} N_{i,k}(u)p_i$ $(u_0 < u < u_m)$
- Degree = k-1, number of control points = n + 1
- Number of knots = m + 1 @ n+ k + 1

→for degree = 1 and number of control points = 4 →(k = 2, n = 3)

→ Number of knots = n + k + 1 = 6

non periodic uniform knot vector (0,0,1,2,3, 3)

* Knot value between 0 and 3 are equally spaced → uniform

Non-periodic (open) uniform B-Spline

- Example
- For curve degree = 3, number of control points = 5
- → $k = 4$, $n = 4$
- → number of knots = $n+k+1 = 9$
- → non periodic knots vector = (0,0,0,0,1,2,2,2)
- For curve degree = 1, number of control points = 5
- → $k = 2$, $n = 4$
- → number of knots = $n + k + 1 = 7$
- → non periodic uniform knots vector = (0, 0, 1, 2, 3, 4, 4)

Non-periodic (open) uniform B-Spline

- For any value of parameters k and n, non periodic knots are determined from

(1.3)

Non-periodic (open) uniform B-Spline

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$$u_i = \begin{cases} 0 & 0 \le i < k \\ i - k + 1 & k \le i \le n \\ n - k + 2 & n < i \le n+k \end{cases} \qquad (1.3)$$

Non-periodic (open) uniform B-Spline

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Non-periodic (open) uniform B-Spline

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Non-periodic (open) uniform B-Spline

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$u = (0, 0, 1, 2, 3, 3)$

B-Spline basis function

$$N_{i,k}(u) = (u - u_i)\frac{N_{i,k-1}(u)}{u_{i+k-1} - u_i} + (u_{i+k} - u)\frac{N_{i+1,k-1}(u)}{u_{i+k} - u_{i+1}} \qquad (1.1)$$

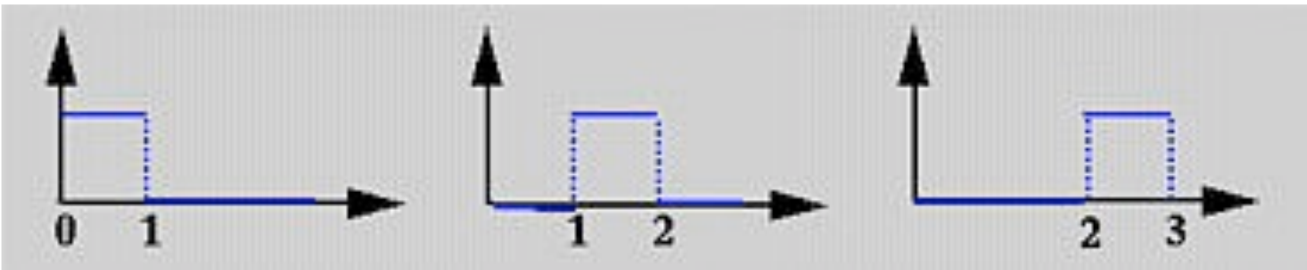
$$N_{i,1} = \begin{cases} 1 & u_i \le u \le u_{i+1} \\ 0 & \text{Otherwise} \end{cases} \qquad (1.2)$$

→In equation (1.1), the denominators can have a value of zero, 0/0 is presumed to be zero.

→If the degree is zero basis function $N_{i,1}(u)$ is 1 if u is in the i-th knot span $[u_i, u_{i+1})$.

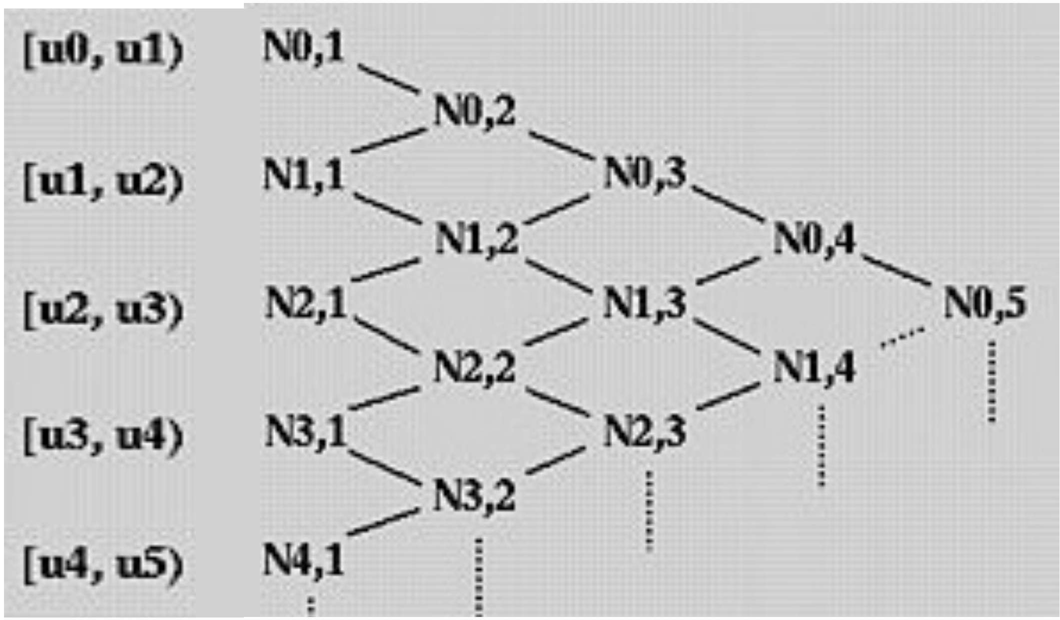
B-Spline basis function

- For example, if we have four knots $u_0 = 0$, $u_1 = 1$, $u_2 = 2$ and $u_3 = 3$, knot spans 0, 1 and 2 are [0,1), [1,2), [2,3)
- the basis functions of degree 0 are $N_{0,1}(u) = 1$ on [0,1) and 0 elsewhere, $N_{1,1}(u) = 1$ on [1,2) and 0 elsewhere, and $N_{2,1}(u) = 1$ on [2,3) and 0 elsewhere.
- This is shown below



B-Spline basis function

- To understand the way of computing $N_{i,p}(u)$ for p greater than 0, we use the triangular computation scheme



Non-periodic (open) uniform B-Spline

Example

- Find the knot values of a non periodic uniform B-Spline which has degree = 2 and 3 control points. Then, find the equation of B-Spline curve in polynomial form.

Non-periodic (open) uniform B-Spline

Non-periodic (open) uniform B-Spline

Answer

Non-periodic (open) uniform B-Spline

Answer

- Degree = k-1 = 2 → k=3

Non-periodic (open) uniform B-Spline

Answer

- Degree = k-1 = 2 → k=3
- Control points = n + 1 = 3 → n=2

Non-periodic (open) uniform B-Spline

Answer

- Degree = k-1 = 2 → k=3
- Control points = n + 1 = 3 → n=2
- Number of knot = n + k + 1 = 6

Non-periodic (open) uniform B-Spline

Answer

- Degree = k-1 = 2 → k=3
- Control points = n + 1 = 3 → n=2
- Number of knot = n + k + 1 = 6
- Knot values → u_0=0, u_1=0, u_2=0, u_3=1,u_4=1,u_5= 1

Non-periodic (open) uniform B-Spline

n

i=0
2

i=0

Non-periodic (open) uniform B-Spline

Answer(cont)

n

$2^{i=0}$

i=0

Non-periodic (open) uniform B-Spline

Answer(cont)

- To obtain the polynomial equation,

  $P(u) = \sum_{i=0}^{n} N_{i,k}(u)p_i$

2

i=0

Non-periodic (open) uniform B-Spline

Answer(cont)

- To obtain the polynomial equation,
  $P(u) = \sum_{i=0}^{n} N_{i,k}(u)p_i$
- $= \sum_{i=0}^{2} N_{i,3}(u)p_i$

Non-periodic (open) uniform B-Spline

Answer(cont)

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- $= \sum_{i=0}^{2} N_{i,3}(u)p_i$
- $= N_{0,3}(u)p_0 + N_{1,3}(u)p_1 + N_{2,3}(u)p_2$

Non-periodic (open) uniform B-Spline

Answer(cont)

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- $\quad = N_{0,3}(u)p_0 + N_{1,3}(u)p_1 + N_{2,3}(u)p_2$

- firstly, find the $N_{i,k}(u)$ using the knot value that shown above, start from k =1 to k=3

Non-periodic (open) uniform B-Spline

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Non-periodic (open) uniform B-Spline

Answer (cont)

Non-periodic (open) uniform B-Spline

Answer (cont)

- For k = 1, find $N_{i,1}(u)$ – use equation (1.2):

Non-periodic (open) uniform B-Spline

Answer (cont)

- For k = 1, find $N_{i,1}(u)$ – use equation (1.2):
  - $N_{0,1}(u) = \begin{cases} 1 & u_0 \leq u \leq u_1 \end{cases}$; (u=0)

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Non-periodic (open) uniform B-Spline

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Non-periodic (open) uniform B-Spline

Answer (cont)

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Non-periodic (open) uniform B-Spline

Answer (cont)

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Non-periodic (open) uniform B-Spline

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Non-periodic (open) uniform B-Spline

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Non-periodic (open) uniform B-Spline

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Non-periodic (open) uniform B-Spline

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Non-periodic (open) uniform B-Spline

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Non-periodic (open) uniform B-Spline

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Non-periodic (open) uniform B-Spline

$$N_{i,k}(u) = (u - u_i)\frac{N_{i,k-1}(u)}{u_{i+k-1} - u_i} + (u_{i+k} - u)\frac{N_{i+1,k-1}(u)}{u_{i+k} - u_{i+1}}$$

Non-periodic (open) uniform B-Spline

Answer (cont)

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Non-periodic (open) uniform B-Spline

Answer (cont)

- For k = 2, find $N_{i,2}(u)$ – use equation (1.1):

$$N_{i,k}(u) = (u - u_i)\frac{N_{i,k-1}(u)}{u_{i+k-1} - u_i} + (u_{i+k} - u)\frac{N_{i+1,k-1}(u)}{u_{i+k} - u_{i+1}}$$

Non-periodic (open) uniform B-Spline

Answer (cont)

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  - $N_{0,2}(u) = \quad \underline{u - u_0}\, N_{0,1} + \underline{u_2 - u}\, N_{1,1} \qquad (u_0 = u_1 = u_2 = 0)$

Non-periodic (open) uniform B-Spline

Answer (cont)

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Non-periodic (open) uniform B-Spline

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Non-periodic (open) uniform B-Spline

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Non-periodic (open) uniform B-Spline

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- $N_{1,2}(u) = \dfrac{u - u_1}{} N_{1,1} + \dfrac{u_3 - u}{} N_{2,1}$ $(u_1 = u_2 = 0, u_3 = 1)$

Non-periodic (open) uniform B-Spline

Answer (cont)

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Non-periodic (open) uniform B-Spline

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  - $u_1 - u_0$ $u_2 - u_1$
  - $= \underline{u - 0}\ N_{0,1} + \underline{0 - u}\ N_{1,1} = 0$
  - $0 - 0$ $0 - 0$
  - $N_{1,2}(u) =$ $\underline{u - u_1}\ N_{1,1} + \underline{u_3 - u}\ N_{2,1}$ $(u_1 = u_2 = 0, u_3 = 1)$
  - $u_2 - u_1$ $u_3 - u_2$
  - $= \underline{u - 0}\ N_{1,1} + \underline{1 - u}\ N_{2,1} = 1 - u$

Non-periodic (open) uniform B-Spline

Answer (cont)

- For k = 2, find $N_{i,2}(u)$ – use equation (1.1):

$$N_{i,k}(u) = (u - u_i)\frac{N_{i,k-1}(u)}{u_{i+k-1} - u_i} + (u_{i+k} - u)\frac{N_{i+1,k-1}(u)}{u_{i+k} - u_{i+1}}$$

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- $= \dfrac{u - 0}{0 - 0} N_{0,1} + \dfrac{0 - u}{0 - 0} N_{1,1} = 0$
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- $= \dfrac{u - 0}{0 - 0} N_{1,1} + \dfrac{1 - u}{1 - 0} N_{2,1} = 1 - u$

Non-periodic (open) uniform B-Spline

Non-periodic (open) uniform B-Spline

Answer (cont)

Non-periodic (open) uniform B-Spline

Answer (cont)

- $N_{2,2}(u) = \dfrac{u - u_2}{} N_{2,1} + \dfrac{u_4 - u}{} N_{3,1}$ $(u_2 = 0,\ u_3 = u_4 = 1)$

Non-periodic (open) uniform B-Spline

Answer (cont)

- $N_{2,2}(u) = \dfrac{u - u_2}{u_3 - u_2} N_{2,1} + \dfrac{u_4 - u}{u_4 - u_3} N_{3,1}$ $(u_2 = 0,\ u_3 = u_4 = 1)$
-

Non-periodic (open) uniform B-Spline

Answer (cont)

- $N_{2,2}(u) = \dfrac{u - u_2}{u_3 - u_2} N_{2,1} + \dfrac{u_4 - u}{u_4 - u_3} N_{3,1}$ $(u_2 = 0,\ u_3 = u_4 = 1)$
- $= \dfrac{u - 0}{} N_{2,1} + \dfrac{1 - u}{} N_{3,1} = u$

Non-periodic (open) uniform B-Spline

Answer (cont)

- $N_{2,2}(u) = \dfrac{u - u_2}{u_3 - u_2} N_{2,1} + \dfrac{u_4 - u}{u_4 - u_3} N_{3,1}$ $\quad (u_2 = 0, u_3 = u_4 = 1)$
- $= \dfrac{u - 0}{1 - 0} N_{2,1} + \dfrac{1 - u}{1 - 1} N_{3,1} = u$

Non-periodic (open) uniform B-Spline

Answer (cont)

- $N_{2,2}(u) = \dfrac{u - u_2}{u_3 - u_2} N_{2,1} + \dfrac{u_4 - u}{u_4 - u_3} N_{3,1}$ $(u_2 = 0,\ u_3 = u_4 = 1)$
- $= \dfrac{u - 0}{1 - 0} N_{2,1} + \dfrac{1 - u}{1 - 1} N_{3,1} = u$
- $N_{3,2}(u) = \dfrac{u - u_3}{} N_{3,1} + \dfrac{u_5 - u}{} N_{4,1}$ $(u_3 = u_4 = u_5 = 1)$

Non-periodic (open) uniform B-Spline

Answer (cont)

- $N_{2,2}(u) = \quad \underline{u - u_2}\, N_{2,1} \; + \underline{u_4 - u}\, N_{3,1} \qquad (u_2 = 0,\ u_3 = u_4 = 1)$
- $\qquad\qquad\quad u_3 - u_2 \qquad\qquad u_4 - u_3$
- $\qquad\qquad = \quad \underline{u - 0}\, N_{2,1} + \quad \underline{1 - u}\, N_{3,1} \; = \; u$
- $\qquad\qquad\quad 1 - 0 \qquad\qquad 1 - 1$
- $N_{3,2}(u) = \qquad\qquad \underline{u - u_3}\, N_{3,1} \; + \underline{u_5 - u}\, N_{4,1} \qquad (u_3 = u_4 = u_5 = 1)$
- $\qquad\qquad\quad u_4 - u_3 \qquad\qquad u_5 - u_4$

Non-periodic (open) uniform B-Spline

Answer (cont)

- $N_{2,2}(u) = \dfrac{u - u_2}{u_3 - u_2} N_{2,1} + \dfrac{u_4 - u}{u_4 - u_3} N_{3,1}$ $(u_2 = 0,\ u_3 = u_4 = 1)$
- $= \dfrac{u - 0}{1 - 0} N_{2,1} + \dfrac{1 - u}{1 - 1} N_{3,1} = u$
- $N_{3,2}(u) = \dfrac{u - u_3}{u_4 - u_3} N_{3,1} + \dfrac{u_5 - u}{u_5 - u_4} N_{4,1}$ $(u_3 = u_4 = u_5 = 1)$
- $= \dfrac{u - 1}{} N_{3,1} + \dfrac{1 - u}{} N_{4,1} = 0$

Non-periodic (open) uniform B-Spline

Answer (cont)

- $N_{2,2}(u) = \quad \underline{u - u_2}\, N_{2,1} \;+\; \underline{u_4 - u}\, N_{3,1} \qquad (u_2 = 0,\ u_3 = u_4 = 1)$
- $\qquad\qquad u_3 - u_2 \qquad\qquad u_4 - u_3$
- $\qquad = \quad \underline{u - 0}\, N_{2,1} + \quad \underline{1 - u}\, N_{3,1} \;=\; u$
- $\qquad\qquad 1 - 0 \qquad\qquad 1 - 1$
- $N_{3,2}(u) = \qquad \underline{u - u_3}\, N_{3,1} \;+\; \underline{u_5 - u}\, N_{4,1} \qquad (u_3 = u_4 = u_5 = 1)$
- $\qquad\qquad u_4 - u_3 \qquad\qquad u_5 - u_4$
- $\qquad = \quad \underline{u - 1}\, N_{3,1} + \quad \underline{1 - u}\, N_{4,1} \;=\; 0$
- $\qquad\qquad 1 - 1 \qquad\qquad 1 - 1$

Non-periodic (open) uniform B-Spline

Answer (cont)

For k = 2

$N_{0,2}(u) = 0$

$N_{1,2}(u) = 1 - u$

$N_{2,2}(u) = u$

$N_{3,2}(u) = 0$

Non-periodic (open) uniform B-Spline

$$N_{i,k}(u) = (u - u_i)\frac{N_{i,k-1}(u)}{u_{i+k-1} - u_i} + (u_{i+k} - u)\frac{N_{i+1,k-1}(u)}{u_{i+k} - u_{i+1}}$$

Non-periodic (open) uniform B-Spline

Answer (cont)

$$N_{i,k}(u) = (u - u_i)\frac{N_{i,k-1}(u)}{u_{i+k-1} - u_i} + (u_{i+k} - u)\frac{N_{i+1,k-1}(u)}{u_{i+k} - u_{i+1}}$$

Non-periodic (open) uniform B-Spline

Answer (cont)

- For k = 3, find $N_{i,3}(u)$ – use equation (1.1):

$$N_{i,k}(u) = (u - u_i)\frac{N_{i,k-1}(u)}{u_{i+k-1} - u_i} + (u_{i+k} - u)\frac{N_{i+1,k-1}(u)}{u_{i+k} - u_{i+1}}$$

Non-periodic (open) uniform B-Spline

Answer (cont)

- For k = 3, find $N_{i,3}(u)$ – use equation (1.1):

$$N_{i,k}(u) = (u - u_i)\frac{N_{i,k-1}(u)}{u_{i+k-1} - u_i} + (u_{i+k} - u)\frac{N_{i+1,k-1}(u)}{u_{i+k} - u_{i+1}}$$

  - $N_{0,3}(u) = \underline{u - u_0}\, N_{0,2} + \underline{u_3 - u}\, N_{1,2}$ $(u_0 = u_1 = u_2 = 0,\ u_3 = 1)$

Non-periodic (open) uniform B-Spline

Answer (cont)

- For k = 3, find $N_{i,3}(u)$ – use equation (1.1):

$$N_{i,k}(u) = (u - u_i)\frac{N_{i,k-1}(u)}{u_{i+k-1} - u_i} + (u_{i+k} - u)\frac{N_{i+1,k-1}(u)}{u_{i+k} - u_{i+1}}$$

- $N_{0,3}(u) = \dfrac{u - u_0}{u_2 - u_0} N_{0,2} + \dfrac{u_3 - u}{u_3 - u_1} N_{1,2}$ $(u_0 = u_1 = u_2 = 0,\ u_3 = 1)$
-

Non-periodic (open) uniform B-Spline

Answer (cont)

- For k = 3, find $N_{i,3}(u)$ – use equation (1.1):

$$N_{i,k}(u) = (u - u_i)\frac{N_{i,k-1}(u)}{u_{i+k-1} - u_i} + (u_{i+k} - u)\frac{N_{i+1,k-1}(u)}{u_{i+k} - u_{i+1}}$$

- $N_{0,3}(u) = \dfrac{u - u_0}{u_2 - u_0} N_{0,2} + \dfrac{u_3 - u}{u_3 - u_1} N_{1,2}$ $\quad (u_0 = u_1 = u_2 = 0,\ u_3 = 1)$
- $= \dfrac{u - 0}{} N_{0,2} + \dfrac{1 - u}{} N_{1,2} = (1-u)(1-u) = (1-u)^2$

Non-periodic (open) uniform B-Spline

Answer (cont)

- For k = 3, find $N_{i,3}(u)$ – use equation (1.1):

$$N_{i,k}(u) = (u - u_i)\frac{N_{i,k-1}(u)}{u_{i+k-1} - u_i} + (u_{i+k} - u)\frac{N_{i+1,k-1}(u)}{u_{i+k} - u_{i+1}}$$

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Non-periodic (open) uniform B-Spline

Answer (cont)

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- $= \frac{u - 0}{0 - 0} N_{0,2} + \frac{1 - u}{1 - 0} N_{1,2} = (1-u)(1-u) = (1-u)^2$
- $N_{1,3}(u) = \frac{u - u_1}{} N_{1,2} + \frac{u_4 - u}{} N_{2,2}$ $(u_1 = u_2 = 0, u_3 = u_4 = 1)$

Non-periodic (open) uniform B-Spline

Answer (cont)

- For k = 3, find $N_{i,3}(u)$ – use equation (1.1):

$$N_{i,k}(u) = (u - u_i)\frac{N_{i,k-1}(u)}{u_{i+k-1} - u_i} + (u_{i+k} - u)\frac{N_{i+1,k-1}(u)}{u_{i+k} - u_{i+1}}$$

  - $N_{0,3}(u) = \frac{u - u_0}{u_2 - u_0} N_{0,2} + \frac{u_3 - u}{u_3 - u_1} N_{1,2}$ $(u_0 = u_1 = u_2 = 0,\ u_3 = 1)$
  - $= \frac{u - 0}{0 - 0} N_{0,2} + \frac{1 - u}{1 - 0} N_{1,2} = (1-u)(1-u) = (1-u)^2$
  - $N_{1,3}(u) = \frac{u - u_1}{u_3 - u_1} N_{1,2} + \frac{u_4 - u}{u_4 - u_2} N_{2,2}$ $(u_1 = u_2 = 0,\ u_3 = u_4 = 1)$

Non-periodic (open) uniform B-Spline

Answer (cont)

- For k = 3, find $N_{i,3}(u)$ – use equation (1.1):

$$N_{i,k}(u) = (u - u_i)\frac{N_{i,k-1}(u)}{u_{i+k-1} - u_i} + (u_{i+k} - u)\frac{N_{i+1,k-1}(u)}{u_{i+k} - u_{i+1}}$$

- $N_{0,3}(u) = \dfrac{u - u_0}{u_2 - u_0} N_{0,2} + \dfrac{u_3 - u}{u_3 - u_1} N_{1,2}$ $(u_0 = u_1 = u_2 = 0,\ u_3 = 1)$
- $= \dfrac{u - 0}{0 - 0} N_{0,2} + \dfrac{1 - u}{1 - 0} N_{1,2} = (1-u)(1-u) = (1-u)^2$
- $N_{1,3}(u) = \dfrac{u - u_1}{u_3 - u_1} N_{1,2} + \dfrac{u_4 - u}{u_4 - u_2} N_{2,2}$ $(u_1 = u_2 = 0,\ u_3 = u_4 = 1)$
- $= \dfrac{u - 0}{1 - 0} N_{1,2} + \dfrac{1 - u}{1 - 0} N_{2,2} = u(1-u) + (1-u)u = 2u(1-u)$

Non-periodic (open) uniform B-Spline

Answer (cont)

- For k = 3, find $N_{i,3}(u)$ – use equation (1.1):

$$N_{i,k}(u) = (u - u_i)\frac{N_{i,k-1}(u)}{u_{i+k-1} - u_i} + (u_{i+k} - u)\frac{N_{i+1,k-1}(u)}{u_{i+k} - u_{i+1}}$$

- $N_{0,3}(u) = \dfrac{u - u_0}{u_2 - u_0} N_{0,2} + \dfrac{u_3 - u}{u_3 - u_1} N_{1,2}$ $(u_0 = u_1 = u_2 = 0,\ u_3 = 1)$
- $= \dfrac{u - 0}{0 - 0} N_{0,2} + \dfrac{1 - u}{1 - 0} N_{1,2} = (1-u)(1-u) = (1-u)^2$
- $N_{1,3}(u) = \dfrac{u - u_1}{u_3 - u_1} N_{1,2} + \dfrac{u_4 - u}{u_4 - u_2} N_{2,2}$ $(u_1 = u_2 = 0,\ u_3 = u_4 = 1)$
- $= \dfrac{u - 0}{1 - 0} N_{1,2} + \dfrac{1 - u}{1 - 0} N_{2,2} = u(1 - u) + (1-u)u = 2u(1-u)$

Non-periodic (open) uniform B-Spline

n

i=0

Non-periodic (open) uniform B-Spline

Answer (cont)

n

i=0

Non-periodic (open) uniform B-Spline

Answer (cont)

- $N_{2,3}(u) = \frac{u - u_2}{\quad} N_{2,2} + \frac{u_5 - u}{\quad} N_{3,2}$ $(u_2 = 0,\ u_3 = u_4 = u_5 = 1)$

n

i=0

Non-periodic (open) uniform B-Spline

Answer (cont)

- $N_{2,3}(u) = \dfrac{u - u_2}{u_4 - u_2} N_{2,2} + \dfrac{u_5 - u}{u_5 - u_3} N_{3,2}$ $(u_2 = 0,\ u_3 = u_4 = u_5 = 1)$
-

n

i=0

Non-periodic (open) uniform B-Spline

Answer (cont)

- $N_{2,3}(u) = \dfrac{u - u_2}{u_4 - u_2} N_{2,2} + \dfrac{u_5 - u}{u_5 - u_3} N_{3,2}$ $(u_2 = 0,\ u_3 = u_4 = u_5 = 1)$
-
- $= \dfrac{u - 0}{} N_{2,2} + \dfrac{1 - u}{} N_{3,2} = u^2$

n

i=0

Non-periodic (open) uniform B-Spline

Answer (cont)

- $N_{2,3}(u) = \dfrac{u - u_2}{u_4 - u_2} N_{2,2} + \dfrac{u_5 - u}{u_5 - u_3} N_{3,2}$ $(u_2 = 0,\ u_3 = u_4 = u_5 = 1)$
- $= \dfrac{u - 0}{1 - 0} N_{2,2} + \dfrac{1 - u}{1 - 1} N_{3,2} = u^2$

n

i=0

Non-periodic (open) uniform B-Spline

Answer (cont)

- $N_{2,3}(u) = \dfrac{u - u_2}{u_4 - u_2} N_{2,2} + \dfrac{u_5 - u}{u_5 - u_3} N_{3,2}$ $\quad (u_2 = 0,\ u_3 = u_4 = u_5 = 1)$
- $= \dfrac{u - 0}{1 - 0} N_{2,2} + \dfrac{1 - u}{1 - 1} N_{3,2} = u^2$

$N_{0,3}(u) = (1 - u)^2,$ $\quad N_{1,3}(u) = 2u(1-u),$ $\quad N_{2,3}(u) = u^2$

n

i=0

Non-periodic (open) uniform B-Spline

Answer (cont)

- $N_{2,3}(u) = \dfrac{u - u_2}{u_4 - u_2} N_{2,2} + \dfrac{u_5 - u}{u_5 - u_3} N_{3,2}$ $(u_2 = 0,\ u_3 = u_4 = u_5 = 1)$
- $= \dfrac{u - 0}{1 - 0} N_{2,2} + \dfrac{1 - u}{1 - 1} N_{3,2} = u^2$

$N_{0,3}(u) = (1 - u)^{2,}$ $N_{1,3}(u) = 2u(1-u),$ $N_{2,3}(u) = u^2$

The polynomial equation, $P(u) = \sum_{i=0}^{n} N_{i,k}(u) p_i$

Non-periodic (open) uniform B-Spline

Answer (cont)

- $N_{2,3}(u) = \dfrac{u - u_2}{u_4 - u_2} N_{2,2} + \dfrac{u_5 - u}{u_5 - u_3} N_{3,2}$ $(u_2 = 0,\ u_3 = u_4 = u_5 = 1)$
- $= \dfrac{u - 0}{1 - 0} N_{2,2} + \dfrac{1 - u}{1 - 1} N_{3,2} = u^2$

$N_{0,3}(u) = (1- u)^2$, $N_{1,3}(u) = 2u(1-u)$, $N_{2,3}(u) = u^2$

The polynomial equation, $P(u) = \sum_{i=0}^{n} N_{i,k}(u) p_i$

- $P(u) = N_{0,3}(u)p_0 + N_{1,3}(u)p_1 + N_{2,3}(u)p_2$

Non-periodic (open) uniform B-Spline

Answer (cont)

- $N_{2,3}(u) = \frac{u - u_2}{u_4 - u_2} N_{2,2} + \frac{u_5 - u}{u_5 - u_3} N_{3,2}$ $(u_2 = 0, u_3 = u_4 = u_5 = 1)$
- $= \frac{u - 0}{1 - 0} N_{2,2} + \frac{1 - u}{1 - 1} N_{3,2} = u^2$

$N_{0,3}(u) = (1 - u)^2,$ $N_{1,3}(u) = 2u(1-u),$ $N_{2,3}(u) = u^2$

The polynomial equation, $P(u) = \sum_{i=0}^{n} N_{i,k}(u)p_i$

- $P(u) = N_{0,3}(u)p_0 + N_{1,3}(u)p_1 + N_{2,3}(u)p_2$
- **$= (1 - u)^2 p_0 + 2u(1-u)\, p_1 + u^2 p_2 \quad (0 <= u <= 1)$**

Non-periodic (open) uniform B-Spline

- Exercise
- Find the polynomial equation for curve with degree = 1 and number of control points = 4

Non-periodic (open) uniform B-Spline

- Answer
- $k = 2$, $n = 3$ → number of knots = 6
- Knot vector = (0, 0, 1, 2, 3, 3)
- For $k = 1$, find $N_{i,1}(u)$ – use equation (1.2):
  - $N_{0,1}(u) = 1 \quad u_0 \leq u \leq u_1$; $(u=0)$
  - $N_{1,1}(u) = 1 \quad u_1 \leq u \leq u_2$; $(0 \leq u \leq 1)$

    $N_{2,1}(u) = 1 \quad u_2 \leq u \leq u_3$; $(1 \leq u \leq 2)$
  - $N_{3,1}(u) = 1 \quad u_3 \leq u \leq u_4$; $(2 \leq u \leq 3)$

    $N_{4,1}(u) = 1 \quad u_4 \leq u \leq u_5$; $(u=3)$
  -

Non-periodic (open) uniform B-Spline

$$N_{i,k}(u) = (u - u_i)\frac{N_{i,k-1}(u)}{u_{i+k-1} - u_i} + (u_{i+k} - u)\frac{N_{i+1,k-1}(u)}{u_{i+k} - u_{i+1}}$$

Non-periodic (open) uniform B-Spline

Answer (cont)

$$N_{i,k}(u) = (u - u_i)\frac{N_{i,k-1}(u)}{u_{i+k-1} - u_i} + (u_{i+k} - u)\frac{N_{i+1,k-1}(u)}{u_{i+k} - u_{i+1}}$$

Non-periodic (open) uniform B-Spline

Answer (cont)

- For k = 2, find $N_{i,2}(u)$ – use equation (1.1):

$$N_{i,k}(u) = (u - u_i)\frac{N_{i,k-1}(u)}{u_{i+k-1} - u_i} + (u_{i+k} - u)\frac{N_{i+1,k-1}(u)}{u_{i+k} - u_{i+1}}$$

Non-periodic (open) uniform B-Spline

Answer (cont)

- For k = 2, find $N_{i,2}(u)$ – use equation (1.1):

$$N_{i,k}(u) = (u - u_i)\frac{N_{i,k-1}(u)}{u_{i+k-1} - u_i} + (u_{i+k} - u)\frac{N_{i+1,k-1}(u)}{u_{i+k} - u_{i+1}}$$

  - $N_{0,2}(u) = \quad \underline{u - u_0}\, N_{0,1} + \underline{u_2 - u}\, N_{1,1} \qquad (u_0 = u_1 = 0,\ u_2 = 1)$

Non-periodic (open) uniform B-Spline

Answer (cont)

- For k = 2, find $N_{i,2}(u)$ – use equation (1.1):

$$N_{i,k}(u) = (u - u_i)\frac{N_{i,k-1}(u)}{u_{i+k-1} - u_i} + (u_{i+k} - u)\frac{N_{i+1,k-1}(u)}{u_{i+k} - u_{i+1}}$$

  - $N_{0,2}(u) = \frac{u - u_0}{u_1 - u_0} N_{0,1} + \frac{u_2 - u}{u_2 - u_1} N_{1,1}$ $(u_0 = u_1 = 0,\ u_2 = 1)$
  -

Non-periodic (open) uniform B-Spline

Answer (cont)

- For k = 2, find $N_{i,2}(u)$ – use equation (1.1):

$$N_{i,k}(u) = (u - u_i)\frac{N_{i,k-1}(u)}{u_{i+k-1} - u_i} + (u_{i+k} - u)\frac{N_{i+1,k-1}(u)}{u_{i+k} - u_{i+1}}$$

  - $N_{0,2}(u) = \dfrac{u - u_0}{u_1 - u_0} N_{0,1} + \dfrac{u_2 - u}{u_2 - u_1} N_{1,1}$ $(u_0 = u_1 = 0,\ u_2 = 1)$
  - $= \dfrac{u - 0}{} N_{0,1} + \dfrac{1 - u}{} N_{1,1}$

Non-periodic (open) uniform B-Spline

Answer (cont)

- For k = 2, find $N_{i,2}(u)$ – use equation (1.1):

$$N_{i,k}(u) = (u - u_i)\frac{N_{i,k-1}(u)}{u_{i+k-1} - u_i} + (u_{i+k} - u)\frac{N_{i+1,k-1}(u)}{u_{i+k} - u_{i+1}}$$

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Non-periodic (open) uniform B-Spline

Answer (cont)

- For k = 2, find $N_{i,2}(u)$ – use equation (1.1):

$$N_{i,k}(u) = (u - u_i)\frac{N_{i,k-1}(u)}{u_{i+k-1} - u_i} + (u_{i+k} - u)\frac{N_{i+1,k-1}(u)}{u_{i+k} - u_{i+1}}$$

- $N_{0,2}(u) = \dfrac{u - u_0}{u_1 - u_0} N_{0,1} + \dfrac{u_2 - u}{u_2 - u_1} N_{1,1}$ $\quad (u_0 = u_1 = 0,\ u_2 = 1)$
- $= \dfrac{u - 0}{0 - 0} N_{0,1} + \dfrac{1 - u}{1 - 0} N_{1,1}$
- $= 1 - u$ $\quad (0 \le u \le 1)$

Non-periodic (open) uniform B-Spline

$$N_{i,k}(u) = (u - u_i)\frac{N_{i,k-1}(u)}{u_{i+k-1} - u_i} + (u_{i+k} - u)\frac{N_{i+1,k-1}(u)}{u_{i+k} - u_{i+1}}$$

Non-periodic (open) uniform B-Spline

Answer (cont)

$$N_{i,k}(u) = (u - u_i)\frac{N_{i,k-1}(u)}{u_{i+k-1} - u_i} + (u_{i+k} - u)\frac{N_{i+1,k-1}(u)}{u_{i+k} - u_{i+1}}$$

Non-periodic (open) uniform B-Spline

Answer (cont)

- For k = 2, find $N_{i,2}(u)$ – use equation (1.1):

$$N_{i,k}(u) = (u - u_i)\frac{N_{i,k-1}(u)}{u_{i+k-1} - u_i} + (u_{i+k} - u)\frac{N_{i+1,k-1}(u)}{u_{i+k} - u_{i+1}}$$

Non-periodic (open) uniform B-Spline

Answer (cont)

- For k = 2, find $N_{i,2}(u)$ – use equation (1.1):

$$N_{i,k}(u) = (u - u_i)\frac{N_{i,k-1}(u)}{u_{i+k-1} - u_i} + (u_{i+k} - u)\frac{N_{i+1,k-1}(u)}{u_{i+k} - u_{i+1}}$$

  - $N_{1,2}(u) = \frac{u - u_1}{} N_{1,1} + \frac{u_3 - u}{} N_{2,1}$ $(u_1 = 0, u_2 = 1, u_3 = 2)$

Non-periodic (open) uniform B-Spline

Answer (cont)

- For k = 2, find $N_{i,2}(u)$ – use equation (1.1):

$$N_{i,k}(u) = (u - u_i)\frac{N_{i,k-1}(u)}{u_{i+k-1} - u_i} + (u_{i+k} - u)\frac{N_{i+1,k-1}(u)}{u_{i+k} - u_{i+1}}$$

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Non-periodic (open) uniform B-Spline

Answer (cont)

- For k = 2, find $N_{i,2}(u)$ – use equation (1.1):

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  - $N_{1,2}(u) =$ $\underline{u - u_1}\ N_{1,1} + \underline{u_3 - u}\ N_{2,1}$ $(u_1 = 0,\ u_2 = 1,\ u_3 = 2)$
  - $u_2 - u_1$ $u_3 - u_2$
  - $= \underline{u - 0}\ N_{1,1} + \underline{2 - u}\ N_{2,1}$

Non-periodic (open) uniform B-Spline

Answer (cont)

- For k = 2, find $N_{i,2}(u)$ – use equation (1.1):

$$N_{i,k}(u) = (u - u_i)\frac{N_{i,k-1}(u)}{u_{i+k-1} - u_i} + (u_{i+k} - u)\frac{N_{i+1,k-1}(u)}{u_{i+k} - u_{i+1}}$$

  - $N_{1,2}(u) = \frac{u - u_1}{u_2 - u_1} N_{1,1} + \frac{u_3 - u}{u_3 - u_2} N_{2,1}$ ($u_1 = 0$, $u_2 = 1$, $u_3 = 2$)
  - $= \frac{u - 0}{1 - 0} N_{1,1} + \frac{2 - u}{2 - 1} N_{2,1}$

Non-periodic (open) uniform B-Spline

Answer (cont)

- For k = 2, find $N_{i,2}(u)$ – use equation (1.1):

$$N_{i,k}(u) = (u - u_i)\frac{N_{i,k-1}(u)}{u_{i+k-1} - u_i} + (u_{i+k} - u)\frac{N_{i+1,k-1}(u)}{u_{i+k} - u_{i+1}}$$

- $N_{1,2}(u) =$ $\underline{u - u_1}\, N_{1,1} + \underline{u_3 - u}\, N_{2,1}$ ($u_1 = 0$, $u_2 = 1$, $u_3 = 2$)
- $u_2 - u_1$ $\quad u_3 - u_2$
- $= \underline{u - 0}\, N_{1,1} + \underline{2 - u}\, N_{2,1}$
- $1 - 0$ $\quad 2 - 1$
- $N_{1,2}(u) = u$ $\quad (0 \leq u \leq 1)$

Non-periodic (open) uniform B-Spline

Answer (cont)

- For k = 2, find $N_{i,2}(u)$ – use equation (1.1):

$$N_{i,k}(u) = (u - u_i)\frac{N_{i,k-1}(u)}{u_{i+k-1} - u_i} + (u_{i+k} - u)\frac{N_{i+1,k-1}(u)}{u_{i+k} - u_{i+1}}$$

- $N_{1,2}(u) = \frac{u - u_1}{u_2 - u_1} N_{1,1} + \frac{u_3 - u}{u_3 - u_2} N_{2,1}$ $\quad (u_1 = 0, u_2 = 1, u_3 = 2)$
- $= \frac{u - 0}{1 - 0} N_{1,1} + \frac{2 - u}{2 - 1} N_{2,1}$
- $N_{1,2}(u) = u \quad (0 \le u \le 1)$
- $N_{1,2}(u) = 2 - u \quad (1 \le u \le 2)$

Non-periodic (open) uniform B-Spline

Non-periodic (open) uniform B-Spline

Answer (cont)

Non-periodic (open) uniform B-Spline

Answer (cont)

- $N_{2,2}(u) = \dfrac{u - u_2}{\;} N_{2,1} + \dfrac{u_4 - u}{\;} N_{3,1}$ $(u_2 = 1, u_3 = 2, u_4 = 3)$

Non-periodic (open) uniform B-Spline

Answer (cont)

- $N_{2,2}(u) = \dfrac{u - u_2}{u_3 - u_2} N_{2,1} + \dfrac{u_4 - u}{u_4 - u_3} N_{3,1}$ $(u_2 = 1,\ u_3 = 2, u_4 = 3)$
-

Non-periodic (open) uniform B-Spline

Answer (cont)

- $N_{2,2}(u) = \dfrac{u - u_2}{u_3 - u_2} N_{2,1} + \dfrac{u_4 - u}{u_4 - u_3} N_{3,1}$ $(u_2 = 1,\ u_3 = 2, u_4 = 3)$
- $= \dfrac{u - 1}{} N_{2,1} + \dfrac{3 - u}{} N_{3,1} =$

Non-periodic (open) uniform B-Spline

Answer (cont)

- $N_{2,2}(u) = \dfrac{u - u_2}{u_3 - u_2} N_{2,1} + \dfrac{u_4 - u}{u_4 - u_3} N_{3,1}$ $\quad (u_2 = 1,\ u_3 = 2, u_4 = 3)$
- $= \dfrac{u - 1}{2 - 1} N_{2,1} + \dfrac{3 - u}{3 - 2} N_{3,1} =$

Non-periodic (open) uniform B-Spline

Answer (cont)

- $N_{2,2}(u) = \dfrac{u - u_2}{u_3 - u_2} N_{2,1} + \dfrac{u_4 - u}{u_4 - u_3} N_{3,1}$ $(u_2 = 1,\ u_3 = 2, u_4 = 3)$
- $= \dfrac{u - 1}{2 - 1} N_{2,1} + \dfrac{3 - u}{3 - 2} N_{3,1} =$
- $N_{2,2}(u) = u - 1 \quad (1 \leq u \leq 2)$

Non-periodic (open) uniform B-Spline

Answer (cont)

- $N_{2,2}(u) = \dfrac{u - u_2}{u_3 - u_2} N_{2,1} + \dfrac{u_4 - u}{u_4 - u_3} N_{3,1}$ $(u_2 = 1, u_3 = 2, u_4 = 3)$
- $= \dfrac{u - 1}{2 - 1} N_{2,1} + \dfrac{3 - u}{3 - 2} N_{3,1} =$
- $N_{2,2}(u) = u - 1 \quad (1 \leq u \leq 2)$
- $N_{2,2}(u) = 3 - u \quad (2 \leq u \leq 3)$

Non-periodic (open) uniform B-Spline

Non-periodic (open) uniform B-Spline

Answer (cont)

Non-periodic (open) uniform B-Spline

Answer (cont)

- $N_{3,2}(u) = \frac{u - u_3}{} N_{3,1} + \frac{u_5 - u}{} N_{4,1}$ $(u_3 = 2, u_4 = 3, u_5 = 3)$

Non-periodic (open) uniform B-Spline

Answer (cont)

- $N_{3,2}(u) = \frac{u - u_3}{u_4 - u_3} N_{3,1} + \frac{u_5 - u}{u_5 - u_4} N_{4,1}$ $(u_3 = 2, u_4 = 3, u_5 = 3)$
-

Non-periodic (open) uniform B-Spline

Answer (cont)

- $N_{3,2}(u) =$ $\underline{u - u_3}$ $N_{3,1}$ $+$ $\underline{u_5 - u}$ $N_{4,1}$ ($u_3 = 2$, $u_4 = 3$, $u_5 =$ 3)
- $u_4 - u_3$ $u_5 - u_4$
- $=$ $\underline{u - 2}$ $N_{3,1}$ $+$ $3\,\underline{-\,u}$ $N_{4,1}$ $=$

Non-periodic (open) uniform B-Spline

Answer (cont)

- $N_{3,2}(u) =$ $\underline{u - u_3}$ $N_{3,1}$ + $\underline{u_5 - u}$ $N_{4,1}$ $(u_3 = 2, u_4 = 3, u_5 = 3)$
- $u_4 - u_3$ $u_5 - u_4$
- $=$ $\underline{u - 2}$ $N_{3,1}$ + $3 \underline{- u}$ $N_{4,1}$ $=$
- $3 - 2$ $3 - 3$

Non-periodic (open) uniform B-Spline

Answer (cont)

- $N_{3,2}(u) =$ $\underline{u - u_3}$ $N_{3,1}$ + $\underline{u_5 - u}$ $N_{4,1}$ ($u_3 = 2$, $u_4 = 3$, $u_5 = 3$)
- $u_4 - u_3$ $u_5 - u_4$
- = $\underline{u - 2}$ $N_{3,1}$ + $3 \underline{- u}$ $N_{4,1}$ =
- $3 - 2$ $3 - 3$
- = $u - 2$ $(2 \leq u \leq 3)$

Non-periodic (open) uniform B-Spline

Non-periodic (open) uniform B-Spline

Answer (cont)

Non-periodic (open) uniform B-Spline

Answer (cont)

- The polynomial equation $P(u) = \sum N_{i,k}(u)p_i$

Non-periodic (open) uniform B-Spline

Answer (cont)

- The polynomial equation $P(u) = \sum N_{i,k}(u)p_i$
- $P(u) = N_{0,2}(u)p_0 + N_{1,2}(u)p_1 + N_{2,2}(u)p_2 + N_{3,2}(u)p_3$

Non-periodic (open) uniform B-Spline

Answer (cont)

- The polynomial equation $P(u) = \sum N_{i,k}(u)p_i$
- $P(u) = N_{0,2}(u)p_0 + N_{1,2}(u)p_1 + N_{2,2}(u)p_2 + N_{3,2}(u)p_3$
- $P(u) = (1 - u)\ p_0 + u\ p_1$ $(0 \leq u \leq 1)$

Non-periodic (open) uniform B-Spline

Answer (cont)

- The polynomial equation $P(u) = \sum N_{i,k}(u)p_i$
- $P(u) = N_{0,2}(u)p_0 + N_{1,2}(u)p_1 + N_{2,2}(u)p_2 + N_{3,2}(u)p_3$

- $P(u) = (1 - u)\, p_0 + u\, p_1$ $(0 \leq u \leq 1)$
- $P(u) = (2 - u)\, p_1 + (u - 1)\, p_2$ $(1 \leq u \leq 2)$

Non-periodic (open) uniform B-Spline

Answer (cont)

- The polynomial equation $P(u) = \sum N_{i,k}(u)p_i$
- $P(u) = N_{0,2}(u)p_0 + N_{1,2}(u)p_1 + N_{2,2}(u)p_2 + N_{3,2}(u)p_3$

- $P(u) = (1 – u)\ p_0 + u\ p_1$ $(0 \le u \le 1)$
- $P(u) = (2 – u)\ p_1 + (u – 1)\ p_2$ $(1 \le u \le 2)$
- $P(u) = (3 – u)\ p_2 + (u - 2)\ p_3$ $(2 \le u \le 3)$

Periodic uniform knot

- Periodic knots are determined from
  - $U_i = i - k \ (0 \le i \le n+k)$
- Example
  - For curve with degree = 3 and number of control points = 4 (cubic B-spline)
  - (k = 4, n = 3) → number of knots = 8
  - (0, 1, 2, 3, 4, 5, 6, 8)

Periodic uniform knot

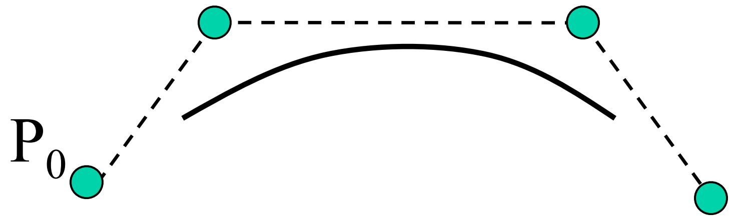
- Normalize u (0<= u <= 1)
- $N_{0,4}(u) = 1/6\ (1-u)^3$
- $N_{1,4}(u) = 1/6\ (3u^3 - 6u^2 + 4)$
- $N_{2,4}(u) = 1/6\ (-3u^3 + 3u^2 + 3u + 1)$
- $N_{3,4}(u) = 1/6\ u^3$

- $P(u) = N_{0,4}(u)p_0 + N_{1,4}(u)p_1 + N_{2,4}(u)p_2 + N_{3,4}(u)p_3$

Periodic uniform knot

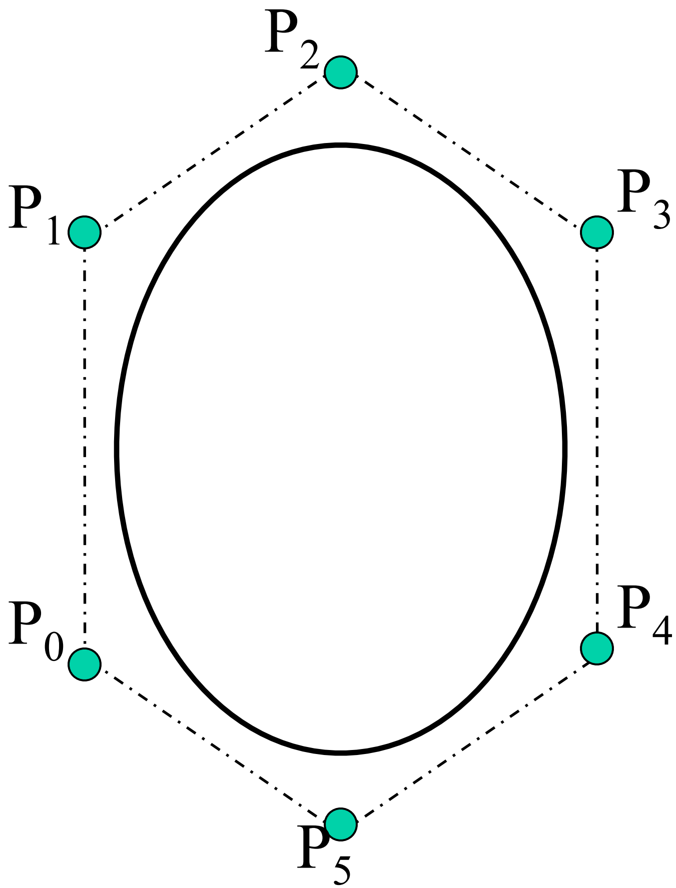
- In matrix form
- $P(u) = [u^3, u^2, u, 1].M_n.\begin{pmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \end{pmatrix}$
- $M_n = 1/6 \begin{pmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 0 & 3 & 0 \\ 1 & 4 & 1 & 0 \end{pmatrix}$

Periodic uniform knot



Closed periodic

Example
k = 4, n = 5



Closed periodic

Equation 1.0 change to

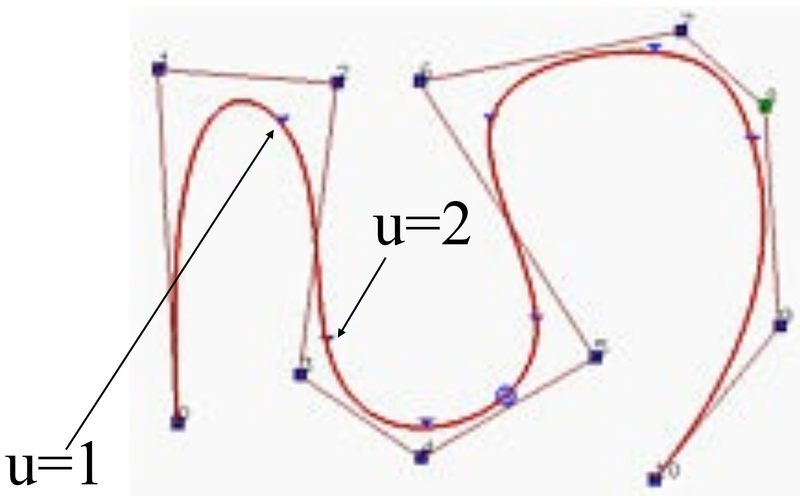
- $N_{i,k}(u) = N_{0,k}((u-i)\bmod(n+1))$

$\rightarrow P(u) = \sum_{i=0}^{n} N_{0,k}((u-i)\bmod(n+1))p_i$

$0 <= u <= n+1$

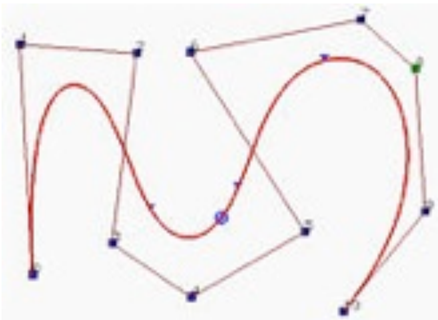
Properties of B-Spline

1. The m degree B-Spline function are piecewise polynomials of degree m → have C^{m-1} continuity. →e.g B-Spline degree 3 have C^2 continuity.

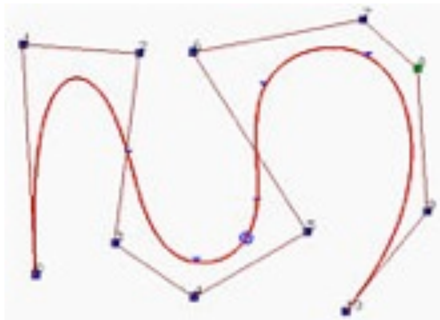


Properties of B-Spline

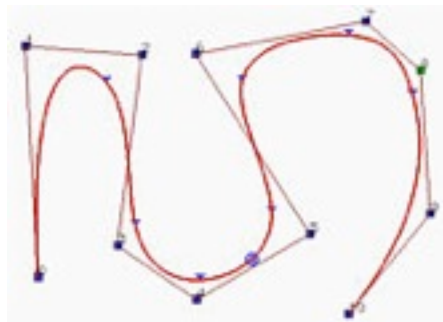
In general, the lower the degree, the closer a B-spline curve follows its control polyline.



Degree = 7



Degree = 5



Degree = 3

Properties of B-Spline

Equality $m = n + k$ must be satisfied

Number of knots = m + 1

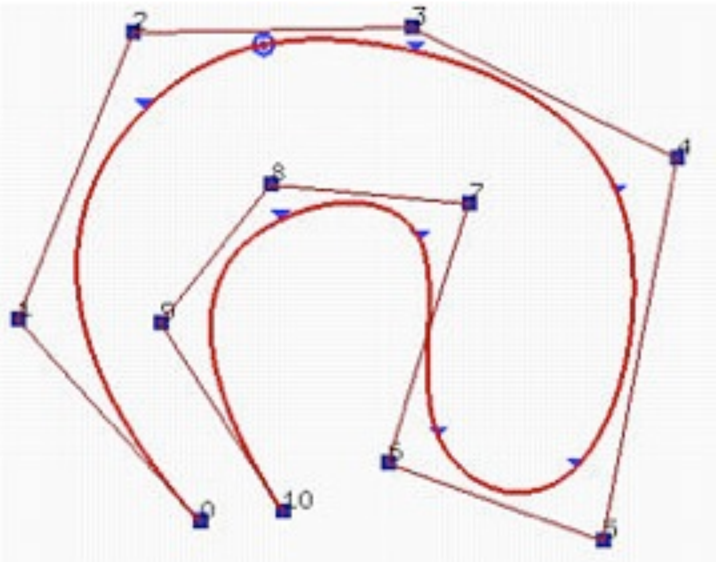
k cannot exceed the number of control points, n+ 1

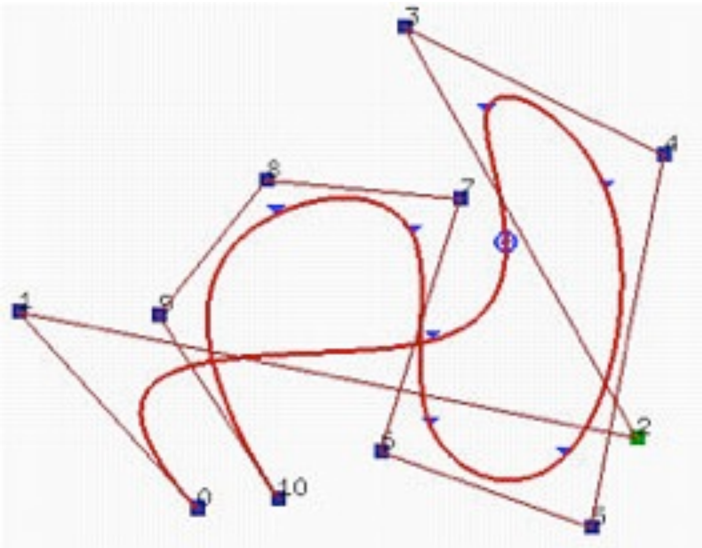
Properties of B-Spline

2. Each curve segment is affected by k control points as shown by past examples.→ e.g k = 3, $P(u) = N_{i-1,k}\, p_{i-1} + N_{i,k}\, p_i + N_{i+1,k}\, p_{i+1}$

Properties of B-Spline

Local Modification Scheme: changing the position of control point P_i only affects the curve $C(u)$ on interval $[u_i, u_{i+k})$.

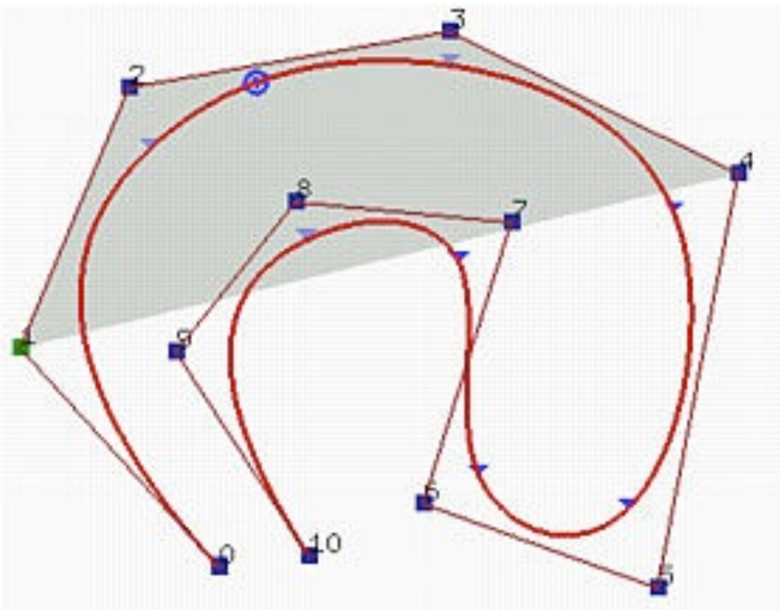




Modify control point P_2

Properties of B-Spline

3. Strong Convex Hull Property: A B-spline curve is contained in the convex hull of its control polyline. More specifically, if u is in knot span $[u_i, u_{i+1})$, then $C(u)$ is in the convex hull of control points P_{i-p}, P_{i-p+1}, ..., P_i.



Degree = 3, k = 4
Convex hull based on 4 control points

Properties of B-Spline

4. Non-periodic B-spline curve C(u) passes through the two end control points P_0 and P_n.
5. Each B-spline function Nk,m(t) is nonnegative for every t, and the family of such functions sums to unity, that is $\sum_{i=0}^{n} N_{i,k}(u) = 1$
6. Affine Invariance

   to transform a B-Spline curve, we simply transform each control points.
7. Bézier Curves Are Special Cases of B-spline Curves

Properties of B-Spline

8. Variation Diminishing : A B-Spline curve does not pass through any line more times than does its control polyline