Supplement to Lecture 17

Global Illumination:
Global Diffuse

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Notes and figures from ***Ed Angel: Interactive Computer Graphics, 6th Ed., 2012 © Addison Wesley***

Global Illumination

- Ray tracing is best with many highly specular sufaces
  - Not characteristic of real scenes
- Rendering equation describes general shading problem
- Radiosity solves rendering equation for perfectly diffuse surfaces

Terminology

- Energy ~ light (incident, transmitted)
  - Must be conserved
- Energy flux = luminous flux = power = energy/unit time
  - Measured in **lumens**
  - Depends on wavelength so we can integrate over spectrum using **luminous efficiency curve** of sensor
- Energy density (Φ) = energy flux/unit area

Terminology (contd).

Intensity ~ brightness

- Brightness is perceptual

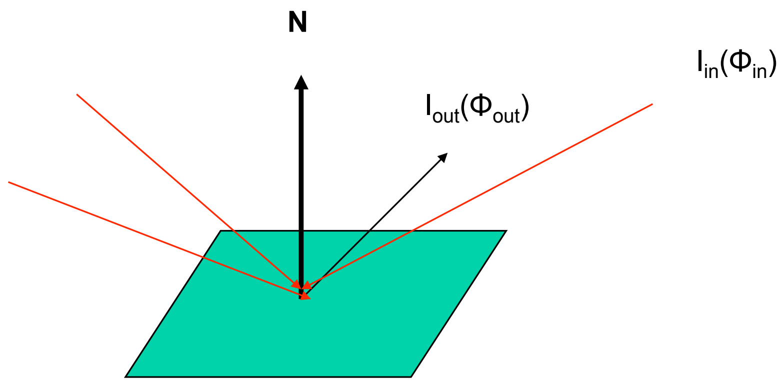
= flux/area-solid angle = power/unit projected area per solid angle

- Measured in **candela**

$$\Phi = \int \int I \, dA \, d\omega$$

Rendering Equation

- Consider a point on a surface



$$I_{out}(\Phi_{out}) = E(\Phi_{out}) + \int_{2\pi} R_{bd}(\Phi_{out}, \Phi_{in}) I_{in}(\Phi_{in}) \cos\theta \, d\omega$$

emission

angle between normal and Φin

bidirectional reflection coefficient

Note that angle is really two angles in 3D and wavelength is fixed

Rendering Equation - 2

- Outgoing light is from two sources
  - Emission
  - Reflection of incoming light
- Must integrate over all incoming light
  - Integrate over hemisphere
- Must account for foreshortening of incoming light

Rendering Equation - 3

- Rendering equation is an energy balance
  - Energy in = energy out
- Integrate over hemisphere
- Fredholm integral equation
  - Cannot be solved analytically in general
- Various approximations of R_{bd} give standard rendering models
- Should also add an occlusion term in front of right side to account for other objects blocking light from reaching surface

Radiosity

- Consider objects to be broken up into flat patches (which may correspond to the polygons in the model)
- Assume that patches are perfectly diffuse reflectors
- Radiosity = flux = energy/unit area/ unit time leaving patch

Notation

n patches numbered 1 to n

b_i = radiosity of patch I

a_i = area patch I

total intensity leaving patch i = $b_i a_i$

$e_i a_i$ = emitted intensity from patch I

ρ_i = reflectivity of patch I

f_{ij} = form factor = fraction of energy leaving patch j that reaches patch i

Modified Integral Equation

Consider light at a point **p** arriving from **p**'

$$i(\mathbf{p}, \mathbf{p}') = \upsilon(\mathbf{p}, \mathbf{p}')(\varepsilon(\mathbf{p},\mathbf{p}') + \int \rho(\mathbf{p}, \mathbf{p}', \mathbf{p}'')i(\mathbf{p}', \mathbf{p}'')d\mathbf{p}''$$

occlusion = 0 or $1/d^2$

emission from **p**' to **p**

light reflected at **p**' from all points **p**'' towards **p**

Radiosity Equation

energy balance

$$b_i a_i = e_i a_i + \rho_i \sum f_{ji} b_j a_j$$

reciprocity

$$f_{ij} a_i = f_{ji} a_j$$

radiosity equation

$$b_i = e_i + \rho_i \sum f_{ij} b_j$$

Matrix Form

$\mathbf{b} = [b_i]$

$\mathbf{e} = [e_i]$

$\mathbf{R} = [r_{ij}]$ $r_{ij} = \rho_i$ if $i \neq j$ $r_{ii} = 0$

$\mathbf{F} = [f_{ij}]$

$$\mathbf{b} = \mathbf{e} + \mathbf{RFb}$$

formal solution

$$\mathbf{b} = [\mathbf{I}-\mathbf{RF}]^{-1}\mathbf{e}$$

$$[\mathbf{I}-\mathbf{RF}]^{-1} = \mathbf{I} + \mathbf{RF} + (\mathbf{RF})^2 + \ldots$$

$$\mathbf{b} = [\mathbf{I}-\mathbf{RF}]^{-1}\mathbf{e} = \mathbf{e} + \mathbf{RFe} + (\mathbf{RF})^2\mathbf{e} + \ldots$$

Solving the Radiosity Equation

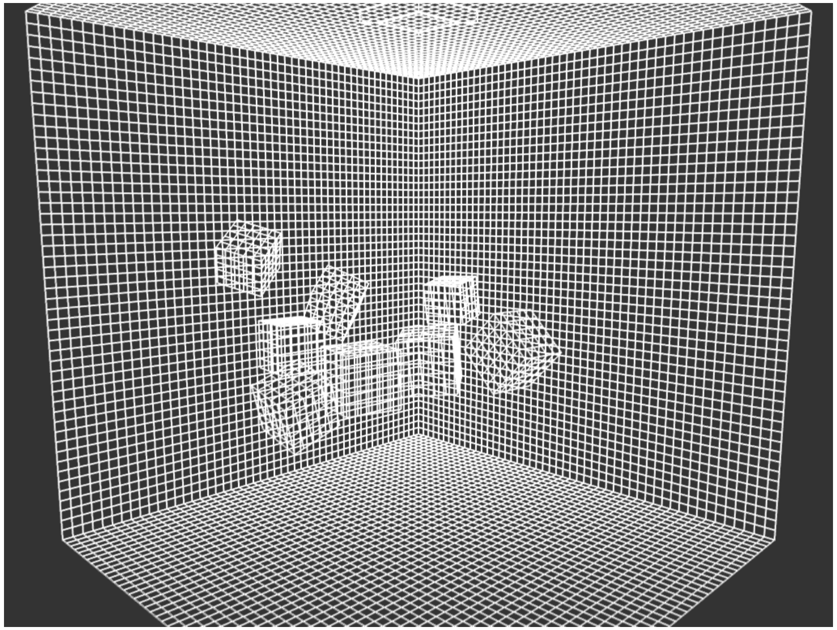
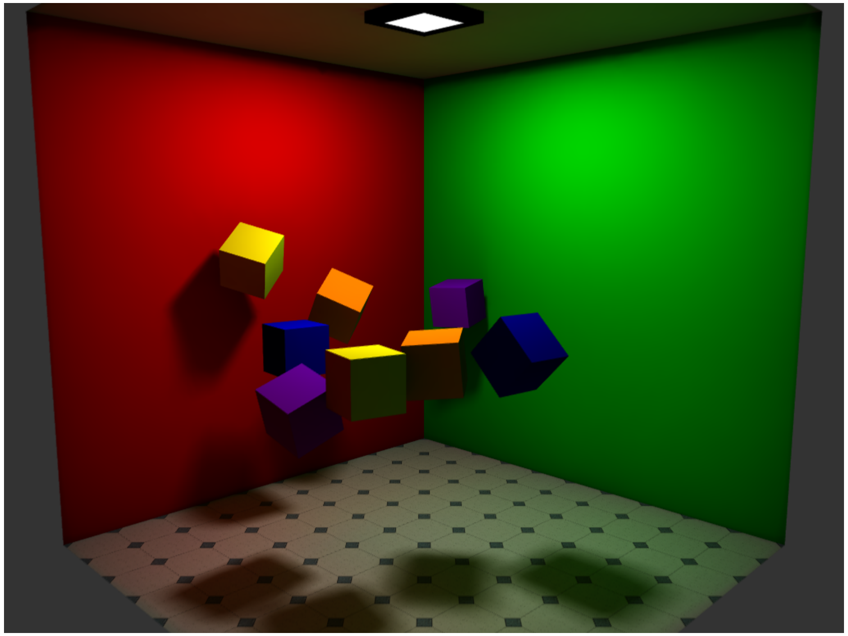
For sparse matrices, iterative methods usually require only O(n) operations per iteration

Jacobi's method

$$\mathbf{b}^{k+1} = \mathbf{e} - \mathbf{RFb}^{k}$$

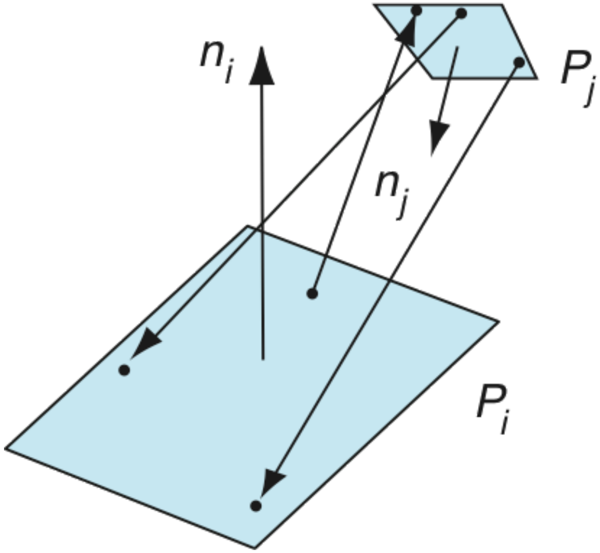
Gauss-Seidel: use immediate updates

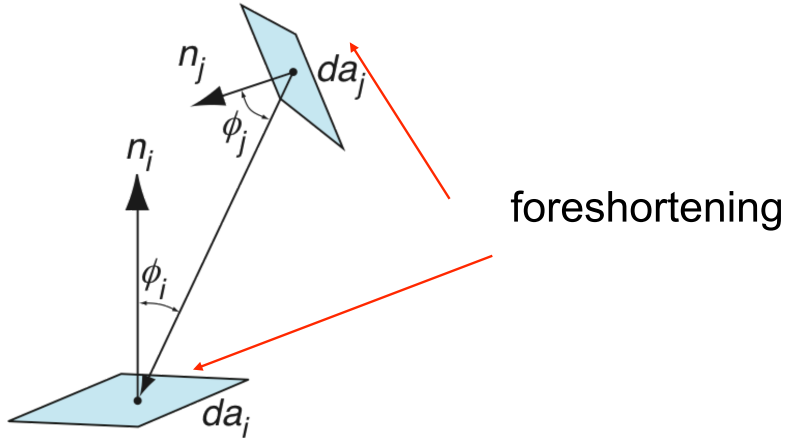
Radiosity Rendered Image

Computing Form Factors

- Consider two flat patches





$$f_{ij} = (1/a_i) \int_{ai} \int_{ai} (o_{ij} \cos \theta_i \cos \theta_j / \pi r^2) da_i\, da_j$$

occlusion

foreshortening of patch j

foreshortening of patch i

Hemisphere/Hemicube