An EWMA Algorithm With a Cycled Resetting (CR) Discount Factor for Drift and Fault of High-Mix Run-to-Run Control

Ying Zheng, Bing Ai, David Shan-Hill Wong, Shi-Shang Jang, Yanwei Wang, and Jie Zhang

Abstract—Run-to-run controllers based on the exponential weighted moving average (EWMA) statistic are probably the most frequently used for the quality control of certain semiconductor manufacturing process steps. The threaded-EWMA run-to-run control is an important stable control scheme. However, the process outputs will deviate largely in the first few runs of each cycle if the disturbance follows an IMA(1,1) series with deterministic linear drift and the thread has a long break length. In this paper, the output of the threaded-EWMA run-to-run control is derived, stability conditions are given, and the causes of large deviations in the first few runs of each cycle are found. Based on the analysis of system performance, a cycled resetting (CR) algorithm for discount factor is proposed to reduce the large deviations, as well as to achieve the minimum asymptotic variance control. Furthermore, how to deal with step fault is also discussed in this paper. By analyzing the influence of the fault, a discount factor resetting fault-tolerant (RFT) approach is proposed. Simulation study shows both the mean square error (MSE) and variance of the output by the proposed algorithm is about 30% to 50% lower than that of the algorithm with fixed discount factor in the process with and without oscillation. This verifies the effectiveness of the proposed approach.

Index Terms—Cycled resetting (CR) algorithm, fault tolerant, threaded-EWMA run-to-run control.

I. INTRODUCTION

S

STATISTICAL process control (SPC) and engineering process control (EPC) are two traditional approaches for process control. SPC attempts to assign a causality relationship to an external disturbance. EPC uses measurements of important process variables to incorporate a feedback loop into the control strategy. Both approaches have sparked a great deal of interest with regard to their potential for application in theory and practice. During the past few years, many researchers and practitioners have looked into combining the advantages of SPC and EPC, thus the run-to-run control approach has been proposed and widely applied in semiconductor manufacturing industry. It is a form of discrete process and machine control in which the product recipe with respect to a particular machine process is modified ex situ, i.e., between machine “runs,” so as to minimize process drift, shift, and variability [1]. Exponentially weighted moving average (EWMA) algorithm is often adopted in run-to-run control. Box and Jenkins [2] carried out pioneering work on the EWMA statistic and showed that EWMA provides the minimum mean square error forecast for an IMA(1,1) process. After Sachs et al. [3] introduced single EWMA into semiconductor manufacturing industry, a considerable number of papers discussed the choices of the optimal EWMA discount factor. Ingolfson and Sachs [4] and Smith [5] analyzed stability and sensitivity of the process output for different closed-loop systems. Butler and Stefani [6] proposed a double EWMA (D-EWMA) controller (termed a predictor-corrector controller) for the system with a deterministic linear drift. Davis et al. [7] and Box [8] proved that the double EWMA filter is a minimum mean square error controller for IMA(2,2) process. Del Castillo [9] analyzed the stability conditions, long-run behavior and transient effects of D-EWMA filter; he also proposed “tradeoff” solution to find the optimal weights to balance long-run variance and transient effect of the D-EWMA filter for the process with disturbance that follows an IMA(1,1) series and a deterministic drift. Tseng et al. [10] extended Del Castillo’s work by using statistical approach to design D-EWMA controller when process disturbance follows an ARIMA(p,d,q) model. Wang and He [11] analyzed and compared the behaviors of single EWMA controller with gain and intercept updating. For the system without drift, Tseng et al. [12] proposed variable EWMA controller to accelerate convergence rate at the beginning of the process.

All the aforementioned models are based on the assumption that there is only a single product manufactured on one tool, which is far from reality. Edgar et al. reviewed the problems of mixed product run-to-run control in a high-mix fab [13] and proposed just-in-time adaptive disturbance estimation (JADE) algorithm to isolate typical types of process disturbance observed in semiconductor industry [14]. Vanli et al. [15] proposed a model selection approach to identify the context variables that contribute most to the process. Ma et al. analyzed tool and product effects in a mixed product and parallel tool environment [16] and proposed ANOVA approach to deal with
run-to-run control of a high mixed operation [17]. Zheng et al. [18] studied a model with two products manufactured on the same tool and proposed two kinds of control method: “tool-based” and “product-based” approaches. They found that the “tool-based” approach is unstable when the plant is non-stationary and the plant-model mismatch parameters are different for different products, while the “product-based” approach, i.e., threaded-EWMA approach is stable. Ai et al. [19] developed this “product-based” approach and Wu et al. [20] gave the further experimental study to prove Zheng’s results.

For the “product-based” approach (see Fig. 1) if the campaign time for one product is long, then other products will have a long break time. For the long thread, although the threaded-EWMA control is stable, the output of other threads will be far deviated from the target at the beginning runs of each cycle (see the example in Section II). These large deviations will bring high rework rate and lots of wastes. However, this problem is considered and solved in this paper.

This paper is organized as follows. In Section II, problem formulation is given, an example is provided to illustrate some problems of such kind of threaded-EWMA approach. In Section III, system performance analysis is taken and cycled resetting (CR) algorithm for discount factor is proposed. Moreover, a step fault is considered, and the discount factor resetting algorithm is introduced in Section IV. In Section V, the simulation study is provided. Conclusion is presented in Section VI. Details of mathematical derivations are given in the Appendices.

II. PROBLEM FORMULATION

In semiconductor manufacturing industry, mixed products are usually produced on the same tool. Consider a simple case that $p$ products are manufactured on a single tool (Fig. 1). The production schedule consists of cycles of $j_{k,t}$ runs in cycle $t$, in which $j_{1,t}$ runs are used to produce product 1, $j_{2,t}$, $j_{3,t}$ runs are used to produce product 2, and $j_{4,t}$, $j_{5,t}$, $j_{6,t}$ runs may have different value for different $t$. $j_{1,t}$ and $i_{1,t}$ are defined as the campaign length and break length for product 1 in cycle $t$, respectively ($i_{1,t}, j_{1,t}, t \in N$).

Assume that the input–output relationship for the products on the given tool is linear with different intercepts $\alpha_1, \alpha_2, \ldots, \alpha_p$ and slopes $\beta_1, \beta_2, \ldots, \beta_p$; all the products share the same tool disturbance $\eta_{c=0}^{i_{1,t}+n}$

$$Y_{c=0}^{i_{1,t}+n} = \begin{cases} \alpha_1 + \beta_1 u_{c=0}^{i_{1,t}+n} + \eta_{c=0}^{i_{1,t}+n}, & 1 \leq n \leq j_{1,t}, \\ \alpha_2 + \beta_2 u_{c=0}^{i_{1,t}+n} + \eta_{c=0}^{i_{1,t}+n}, & j_{1,t} < n \leq j_{1,t} + j_{2,t}, \\ \vdots \\ \alpha_p + \beta_p u_{c=0}^{i_{1,t}+n} + \eta_{c=0}^{i_{1,t}+n}, & j_{p,t} < n \leq i_{1,t} \\ \end{cases}$$

where $u_{c=0}^{n}$ is the manipulated variable at the beginning of the run, $Y_{c=0}^{i_{1,t}+n}$ is the output of product 1 at the end of the run, $Y_{c=0}^{i_{1,t}+n} = \sum_{c=0}^{n-1} \alpha_i + \beta_i u_{c=0}^{i_{1,t}+n} + \eta_{c=0}^{i_{1,t}+n}$

is the output of product 1 at the end of the run, $Y_{c=0}^{i_{1,t}+n} = \sum_{c=0}^{n-1} \alpha_i + \beta_i u_{c=0}^{i_{1,t}+n} + \eta_{c=0}^{i_{1,t}+n}$

and $\theta$ is a parameter of the model. $\eta_{c=0}^{i_{1,t}+n}$ is the output of product 2 at the end of the run, and so on.

In this paper, assume the process disturbance $\{\eta_k\}$ follows an IMA (1, 1) series with deterministic linear drift $\delta$, i.e.,

$$\eta_k = \eta_{k-1} + \delta$$

where $\theta$ is the parameter of IMA(1,1) and $|\theta| \leq 1$; $\varepsilon_k$ is independent identically distributed random variables and $\varepsilon_k \sim N(0, \sigma^2)$. IMA is the abbreviation of integrated moving average, and the IMA $(d, q)$ is a moving average with $q$ parameters which has been integrated $d$ times.

In the threaded run-to-run control, the EWMA filter action is performed with respect to the last run on which the same product is processed. Thus, the output of product 1 in this approach is irrelevant with what is produced in other runs, and the process in which product 1 is manufactured is called thread 1. Also, the process in which product $p$ is manufactured is called thread $p$. Without loss of generality, the behavior of product 1 will only be focused on in the following sections, and the next cases are just a generalization of the product 1’s case.

The process predicted model of product 1 is

$$\hat{Y}_{c=0}^{i_{1,t}+n} = \tilde{a}_1 + b_1 u_{c=0}^{i_{1,t}+n}, \quad n = 1, 2, \ldots, j_{1,t}, \quad \tilde{a}_1$$

and $b_1$ are model offset and gain parameters respectively.

Using the threaded-EWMA filter, the process offset is estimated as shown in equation (4) at the bottom of the next page, the deadheat controller is

$$u_{c=0}^{i_{1,t}+n} = \begin{cases} \frac{\tilde{a}_1 - \tilde{a}_3}{b_1}, & n = 1 \quad t = 0 \\ \frac{T_1 - \tilde{a}_3}{b_1}, & \begin{cases} \frac{T_1 - \tilde{a}_3}{b_1}, & t \geq 1 \end{cases} \end{cases}$$

where $T_1$ is the desired target of product 1 and $\lambda_1 \in (0, 1]$ is called discount factor.

In following example, assume that two products are manufactured on the same tool for 5 cycles, each cycle has the same length, i.e., $i_0 = i_1 = i_2 = i_3 = i_4 = 200$, and the campaign lengths for product 1 in each cycle also have the same value, that is $j_{1,t} = 100$, for $t \in [0, 4]$. Set the parameter of product 1 to be $\alpha_1 = 2$, $\beta_1 = 2$, $\theta = 0.5, \sigma^2 = 0.12$, $\delta = 0.1$. Also, the least square estimate (LSE) of $(\alpha_1, \beta_1)$ is $(\tilde{a}_1, b_1) = (0, 1)$ and the target value of product 1 is $T_1 = 0$. Fig. 2 shows the outputs of product 1 by using threaded-EWMA controller. It is interesting to see that when discount factor $\lambda_1 = 0.9$, the simulated process output oscillates, as shown in Fig. 2(a), but in Fig. 2(b) when $\lambda_1 = 0.2$, the output of the system does not oscillate. For both cases, it is clear that at the beginning runs of each cycle, especially for $t = 1, 2, 3, 4$, the process outputs are far deviated from the target, but considerable runs later, the outputs hit the
and satisfies (2). The following production cycle.

Fig. 1. An example of \( p \) products are manufactured on the same tool in the \( t \)th and \( (t+1) \)th production cycle.

target with a small bias. In other words, the output converges to the desired target very slowly, which will result in a high rework rate.

Modern semiconductor manufacturing is an industry with high cost. Improvement in production efficiency will potentially be very beneficial to manufacturers. In the following sections, several following questions raised in above example will be answered, i.e.,

1) Under what condition the oscillation will happen or not happen?
2) Why the process outputs are far deviated from the target in the first few runs of each cycle?
3) How to reduce the large devation at the beginning runs of each cycle? i.e., how to accelerate the convergence rate?
4) How to deal with a step fault?

III. PROCESS PERFORMANCE ANALYSIS BASED ON THREADED-EWMA APPROACH

In this section, stability condition which based on the threaded-EWMA run-to-run control is given. The cause of large deviations in the first few runs of each cycle is found by the analysis of the system outputs. Furthermore, CR algorithm for discount factor is proposed to reduce the large deviations at the beginning runs of each cycle, as well as to obtain the minimum asymptotic variance control.

A. Stability Analysis

In this subsection, the influence of model mismatch and discount factor on the stability of the threaded-EWMA run-to-run control scheme will be examined.

Since the disturbance \( \{\eta_k\} \) satisfies (2). The following lemmas give the output of product 1.

\[
Y_n = \varphi_n^{(-1)}(\alpha_1 + \xi_1 T_1 - \xi_1 \bar{a}_1 + \bar{e} + \delta) + (1 - \varphi_n^{(-1)})T_1 + \sum_{k=0}^{n-2} \varphi_k^T(\varepsilon_{n-k} - \theta \varepsilon_{n-k-1} + \delta) \tag{6}
\]

with \( \xi_1 = \beta_1/b_1 \) is the model mismatch for product 1, and \( \varphi_1 = 1 - \xi_1 \lambda_1 \) is the output growth factor for product 1.

\begin{proof}
See Appendix A for details.
\end{proof}

Lemma 2: The output of product 1 at each run in the \( t \)th \((t \geq 1)\) production cycle is given in equation (7) at the bottom of the next page with

\[
\chi_2 = \begin{cases} 0, & t < 2 \\ 1, & t \geq 2 \end{cases}.
\]

\begin{proof}
See Appendix B. for details.
\end{proof}
Lemma 3: If $|\phi_1| < 1$, then the asymptotic variance ($AVar$) of product 1’s output in cycle $t$ ($t \geq 0$) is
\[
AVar(Y_n) = \lim_{n \to \infty} Var(Y_n) = \left( \frac{\phi_1^2 - 2\phi_1\theta + \theta^2}{1 - \phi_1^2} \right) \sigma^2 (8)
\]
for $t = 0$
\[
AVar \left( \sum_{i=0}^{\infty} i \varphi_i \right) = \lim_{n \to \infty} Var \left( \sum_{i=0}^{n} i \varphi_i \right) = \frac{\phi_1^2 - 2\phi_1\theta + \theta^2}{1 - \phi_1^2} \sigma^2 (9)
\]
for $t \geq 1$.

Proof: See Appendix C for details.

Again, if $|\phi_1| < 1$, then the mathematic expectation of product 1’s output at the $n_{th}$ run in cycle $t$ ($t \geq 0$) is
\[
E(Y_n) = \phi_1^{n-1}(\alpha_1 + \xi T) + (1 - \phi_1^{n-1})T + \frac{1 - \phi_1^T}{1 - \phi_1} \delta (10)
\]
for $t = 0$ and
\[
E \left( \sum_{i=0}^{n} i \varphi_i \right) = T + \frac{\delta}{1 - \phi_1} + \phi_1^{n-1}(i_{n-1} - j_{n-1}) \delta (11)
\]
for $t \geq 1$.

Combine (8)–(11), the following theorem can be obtained.

Theorem 1: (Stability Condition for Fixed Discount Factor): If $|\phi_1| < 1$, then the system with fixed discount factor EWMA controller is stable.

Proof: If $|\phi_1| < 1$, it is clear that $AVar(\sum_{i=0}^{n} i \varphi_i) < \infty$ combining (8) and (9); on the other hand, one can easily concluded from (10) and (11) that $\lim_{n \to \infty} E \left( \sum_{i=0}^{n} i \varphi_i \right) = T_1 + \delta/(1 - \phi_1)$. Therefore, Theorem 1 holds.

Corollary: When discount factor $0 < \lambda_1 \leq 1/\xi_1$, the process is stable and the response is overdamped; if $1/\xi_1 < \lambda_1 < 2/\xi_1$, the process is also stable but with oscillatory behavior.

Proof: When $\phi_1 \geq 0$, one gets $E \left( \sum_{i=0}^{n} i \varphi_i \right) \geq T_1 + \delta/(1 - \phi_1)$ from (10) and (11), and from Theorem 1, one can conclude that the process is stable and without oscillatory behavior if $0 \leq \phi_1 < 1$, i.e., $0 < \lambda_1 \leq 1/\xi_1$. Also, from (10) and (11), when $-1 < \phi_1 < 0$, one gets $E \left( \sum_{i=0}^{n} i \varphi_i \right) > T_1 + \delta/(1 - \phi_1)$ if $n$ is odd, $E \left( \sum_{i=0}^{n} i \varphi_i \right) < T_1 + \delta/(1 - \phi_1)$ if $n$ is even. So, if $-1 < \phi_1 < 0$, i.e., $1/\xi_1 < \lambda_1 < 2/\xi_1$, then the process is stable but with oscillatory behavior.

For overestimate of process gain, i.e., $\xi_1 \leq 1$, since $0 < \lambda_1 \leq 1$, $\lambda_1$ can only lies in $[0, 1/\xi_1]$ which leads the performance of the process overdamped. On the other hand, for underestimate of
process gain, i.e., \( \xi_1 > 1 \). \( \lambda_1 \) possibly belongs to \((1/\xi_1, 2/\xi_1)\), i.e., the process is likely to oscillate.

### B. System Output

As mentioned in Section II, the process output is far deviated from the desired target at the beginning runs of cycle \( \left( \sum_{c=0}^{t-1} \delta c + 1 \right)_{th} \), after a considerable runs, the outputs hit the target with a small bias. In this subsection, the main reason for this problem will be given.

For the stable process, the following lemma will be obtained by combining (10) and (11).

**Lemma 4:** The main biases of product 1’s outputs between the \( \left( \sum_{c=0}^{t-1} \delta c + 1 \right)_{th} \) and \( \left( \sum_{c=0}^{t-1} \delta c + n - 1 \right)_{th} \) runs, the \( \left( \sum_{c=0}^{t-1} \delta c + n - 1 \right)_{th} \) and \( \left( \sum_{c=0}^{t-1} \delta c + n - 1 \right)_{th} \) runs are

\[
E \left( Y_{t-1} \sum_{c=0}^{\delta c + 1} \right) - E \left( Y_{t-1} \sum_{c=0}^{\delta c + \delta_1 t - 1} \right) = (\delta_{t-1} - \delta_{t-1}) \delta \tag{12}
\]

\[
E \left( Y_{t-1} \sum_{c=n}^{\delta c + n} \right) - E \left( Y_{t-1} \sum_{c=n}^{\delta c + n - 1} \right) = \varphi_1^{n-2}(\varphi_1 - 1)(\delta_{t-1} - \delta_{t-1}) \delta \tag{13}
\]

where \( t \geq 1 \) and \( n \geq 2 \).

**Proof:** It can be derived from (10) and (11) directly.

**Remark 1:** From (10), one can conclude that the process output in cycle 0 will hit the target with a bias of \( \delta / \xi_1 \lambda_1 \) after a few runs of production.

**Remark 2:** If the thread 1 has a long break in the \( (t - 1)_{th} \) production cycle, i.e., \( \delta_{t-1} - \delta_{t-1} \) is large where \( t \geq 1 \), since \( [\varphi_1] < 1 \), one can conclude from (11) that in the \( t_{th} \) cycle the process output will also hit the target with a bias of \( \delta / \xi_1 \lambda_1 \) after a few runs of production.

**Remark 3:** From (12) and (13), it is obvious that at the first few runs of cycle \( t \geq 1 \) the bias of process output will be large when there is a long break in the \( (t - 1)_{th} \) production cycle.

**Remark 4:** Equation (13) shows that the difference of process outputs between \( n_{th} \) and \( (n - 1)_{th} \) run is smaller than that of the previous two runs in the same cycle.

### C. Cycled Resetting (CR) Algorithm for Discount Factor

Theoretical analysis in Section III-B is consistent with simulation results in Fig. 2, i.e., it will take a lot of runs for the general threaded-EWMA controller (with fixed discount factor) to eliminate those large deviations which will bring high rework rate and poor efficiency. In order to handle those problems, CR algorithm for discount factor is proposed.

**Lemma 5:** To achieve minimum asymptotic variance control of product 1, discount factor should be \( \lambda_1 = (1 - \theta) / \xi_1 \).

**Proof:** See Appendix D for the procedure.

**Theorem 2:** (CR Algorithm for Discount Factor) For a stable process, discount factor of product 1 in cycle \( t \) at the \( n_{th} \) run should be

\[
\lambda_1 \left( \sum_{c=0}^{t-1} \delta c + n \right) = \begin{cases} 
\lambda_1^1 + g_1 \left( \sum_{c=0}^{t-1} \delta c + n \right), & 0 < \lambda_1^1 \leq \frac{1}{\xi_1} \\
\lambda_1^1 - g_1 \left( \sum_{c=0}^{t-1} \delta c + n \right), & \frac{1}{\xi_1} < \lambda_1^1 < \frac{2}{\xi_1} 
\end{cases}
\]

(14)

where \( g_1 \left( \sum_{c=0}^{t-1} \delta c + n \right) \) is a decreasing function of \( n \) and satisfies \( \forall n, g_1 \left( \sum_{c=0}^{t-1} \delta c + n \right) \geq 0, 0 < \lambda_1^1 + g_1 \left( \sum_{c=0}^{t-1} \delta c + n \right) \leq 1/\xi_1 \) and \( 1/\xi_1 < \lambda_1^1 - g_1 \left( \sum_{c=0}^{t-1} \delta c + n \right) < 2/\xi_1 \).

**Proof:** See Appendix E for details.

Since in the first few runs of each cycle, the process outputs are far out of specification especially for cycle \( t \geq 1 \), at this time more emphasis should be given to the large deviations; with the increase of \( n \), the bias becomes smaller, at this time the minimum variance control should be considered.

### IV. Fault-Tolerate Control

In semiconductor manufacturing industry, there are many events which can result in immediate faults. Generally, the exact time when fault happens is seldom known, and the fault will bring inevitable oscillations and destructions to the process. Therefore, the controller should learn from the system outputs and try to handle the fault.

In practical manufacturing process, the tool usually undergoes a maintenance event, such event usually causes a shift which can be seen as a step fault to the process. In this section, based on the analysis of the process which experiences such a step fault, discount factor resetting fault-tolerant (RFT) approach is proposed.

If it happens at \( h_{th} \) run, the step fault is denoted as (15)

\[
f_s = \begin{cases} 
f, & s \geq h \\
0, & s < h
\end{cases}
\]

(15)

where \( f \) is the magnitude of the fault.

If \( F_s \) denotes the summation of tool disturbance \( (\eta_s) \) and the step fault \( (f_s) \) at the \( s_{th} \) run, i.e.,

\[
F_s = \eta_s + f_s.
\]

(16)

Then, the following lemmas hold.

**Lemma 6:** When a step fault happens, the mathematic expectation of product 1’s output at the \( n_{th} \) run in cycle 0 is

\[
E(Y_n) = (1 - \varphi_1^{-1})T_1 + \varphi_1^{-1}(\varphi_1 + \xi_1 T_1 - \xi_1 \varphi_1)
+ \frac{1 - \varphi_1^{-1}}{1 - \varphi_1} \sum_{k=2}^{n} \varphi_1^{n-k} \left( f_k - f_{k-1} \right) + \varphi_1^{n-1} f_1.
\]

(17)

**Proof:** See Appendix F for details.
Lemma 7: When a step fault happens, the mathematic expectation of product 1’s output at the nth cycle in cycle \( t (t \geq 1) \) can be described as

\[
E \left( Y_{t-1} \sum_{c=0}^{n} i_{c+n} \right) \\
= T_1 + \frac{\delta}{1 - \varphi_1} + \varphi_1^{n-1} (i_{t-1} - j_{1,t-1}) \delta \\
+ \varphi_1^{n-1} \left( \sum_{c=0}^{n} i_{c+1} + \sum_{c=0}^{n} i_{c+j_{1,t-1}} \right) \\
+ \varphi_1^{n-1} \sum_{k=2}^{n} \varphi_1^{j_{1,t-1} - k} \left( \frac{f_{r-1}}{\sum_{c=0}^{n} i_{c+k}} - \frac{f_{r-1}}{\sum_{c=0}^{n} i_{c+k+1}} \right) \\
+ \gamma_2 \sum_{k=2}^{n} \varphi_1^{j_{1,t-1} - k} \left( \frac{f_{r-1}}{\sum_{c=0}^{n} i_{c+k}} - \frac{f_{r-1}}{\sum_{c=0}^{n} i_{c+k+1}} \right).
\]  

(18)

Proof: See Appendix G for details.

From (17) and (18), one can conclude that if the fault happens at the \( h_{th} \) run, the step fault will lead the process \( f \) away from target at the \( h_{th} \) run. By means of mathematical induction, one can easily conclude that this fault will lead the process \( \varphi_1^{n} \cdot f \) away from target at the \( (h+s)_{th} \) run. In order to overcome the fault, Theorem 3 is proposed.

Theorem 3: (Discount Factor Resetting Fault-Tolerant Approach) If a step fault happens at the \( h_{th} \) run in cycle \( t \), then the discount factor at the \( (h+s)_{th} \) run in Theorem 2, i.e., \( \lambda_1 \left( \sum_{c=0}^{n} i_{c+h+s} \right) \), should be reset as \( \lambda_1 \left( \sum_{c=0}^{n} i_{c} \right) \) to compensate the deviation caused by the step fault. The problem of how to choose a small \( \left| \varphi_1 \right| \) is transformed into another question that is how to choose discount factor \( \lambda_1 \left( \sum_{c=0}^{n} i_{c} \right) \). Since it has been discussed in Theorem 2, it is omitted here.

V. SIMULATION STUDY

In this section, some simulation examples are provided to illustrate the arguments presented in previous sections. Without loss of generality, the following examples will only focus on two products manufactured on the same tool with different cycle length, i.e., different \( i_t \) for different \( t \). The campaign length for product 1, i.e., \( j_{1,t} \), can also be different for different \( t \). Assume that other parameters are \( \sigma^2 = 0.1^2 \), \( \delta = 0.1 \), target values of product 1 and 2 are \( (T_1, T_2) \) respectively.

In the simulation of Figs. 3–6, production cycle is assumed to be \( 4 \), i.e., \( t \in [0, 3] \), the length of each cycle and the campaign length for product 1 in each cycle are \( i_t = 200 \) and \( j_{1,t} = 100 \) for \( \forall t \in [0, 3] \), respectively.

Fig. 3 shows simulated outputs of products 1 and 2 in cycles 0–3, with \( (\xi_1, \xi_2) = (0.5, 4) \), and \( (\lambda_1, \lambda_2) = (0.1, 0.505) \).
### Table I

<table>
<thead>
<tr>
<th></th>
<th>Cycle 0 The decrease percentage</th>
<th>Cycle 1 The decrease percentage</th>
<th>Cycle 2 The decrease percentage</th>
<th>Cycle 3 The decrease percentage</th>
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</tr>
<tr>
<td>CR-variance2</td>
<td>1.0180</td>
<td>39.7%</td>
<td>1.1159</td>
<td>41.9%</td>
</tr>
</tbody>
</table>

### Table II

<table>
<thead>
<tr>
<th></th>
<th>Cycle 0 The decrease percentage</th>
<th>Cycle 1 The decrease percentage</th>
<th>Cycle 2 The decrease percentage</th>
<th>Cycle 3 The decrease percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>fixed-MSE1</td>
<td>0.1030</td>
<td>1.4226</td>
<td>2.2773</td>
<td>1.9916</td>
</tr>
<tr>
<td>CR-MSE1</td>
<td>0.0652</td>
<td>36.7%</td>
<td>0.7864</td>
<td>44.7%</td>
</tr>
<tr>
<td>fixed-MSE2</td>
<td>5.9615</td>
<td>2.1473</td>
<td>2.7611</td>
<td>0.7785</td>
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<td>CR-MSE2</td>
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<td>1.1799</td>
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</tr>
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<td>fixed-variance1</td>
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<td>CR-variance1</td>
<td>0.0584</td>
<td>40.2%</td>
<td>0.7757</td>
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<tr>
<td>fixed-variance2</td>
<td>5.9978</td>
<td>2.1531</td>
<td>2.7719</td>
<td>0.7784</td>
</tr>
<tr>
<td>CR-variance2</td>
<td>3.2053</td>
<td>46.6%</td>
<td>1.1665</td>
<td>45.8%</td>
</tr>
</tbody>
</table>

Fig. 5: Output of products 1 and 2 in cycles 0–3 with fixed and CR discount factor of the process without oscillation.

Fig. 6: Output of products 1 and 2 in cycles 0–3 with fixed and CR discount factor of the process with oscillation.

Fig. 5 is the simulated outputs of product 1 and 2 in cycle 0–3 with fixed discount factor and proposed CR discount factor. In this simulation, \( \theta = 0.7 \) and model mismatches \((\xi_1, \xi_2) = (2, 0.5)\). From Lemma 5, one can conclude that \( \lambda_1^* = 0.15 \) and \( \lambda_2^* = 0.6 \), both of which will lead the system stable and nonoscillatory. If \( \lambda_1 = 0.15 \) as fixed discount factor and \( \lambda_1 \left( \sum_{i=0}^{t-1} j_i + n \right) = 0.15 + 0.3^t \) as the CR discount factor for thread 1; \( \lambda_2 = 0.6 \) and \( \lambda_2 \left( \sum_{i=0}^{t-1} j_i + n \right) = 0.6 + 0.75^{t-n} \) as fixed and CR discount factors for thread 2, it is obvious in Fig. 5 that the process with CR discount factor performs better than the process with fixed discount factor. Table I gives the comparison...
The product 1 here is low-running, both respectively. From Fig. 6, it is clear that the output of both threads oscillate at the beginning runs of each cycle, but the threads with CR discount factors perform better than the threads with fixed discount factors. Detail comparisons of the performances are shown in Table II.

If the production cycle has a variable length, i.e., \( i_t \) has different value for different \( t \), and the campaign lengths for product 1 also have different values. Then, Fig. 7 is the simulation of such case. In this simulation, products 1 and 2 are manufactured on the same tool for 4 cycles, with \( i_0 = 200, i_1 = 400, i_2 = 300, \) and \( i_3 = 500 \). The product 1 here is low-running product with campaign lengths \( j_{1,0} = 10, j_{1,1} = 15, j_{1,2} = 10, \) and \( j_{1,3} = 20 \). Fig. 7(a) and (b) are simulated results for the output without and with oscillation. Other parameters are the same as those used in Figs. 5 and 6. From the figure, it is obvious that both products by CR algorithm outperform those by fixed discount factor. Also, one knows that the larger campaign length in cycle \( t - 1 \), the larger deviations at the beginning runs of cycle \( t \). For example, in the first cycle the campaign length for product 1 is 385, then, in the first few runs of cycle 2 the outputs of product 1 are far deviated from the target value. This result is consistent with theoretical results in Section III. Detailed comparisons of low-running product’s performances by CR discount factor and fixed discount factor are listed in Table III.

If a step fault happens, the performances of products 1 and 2 are illustrated in Fig. 8. Fig. 8(a) shows the process disturbance which follows an IMA(1,1) series with deterministic linear drift and a step fault with a magnitude of 25 which happens at the 250th run. The outputs of overdamped system with fixed and RFT discount factor approaches are given in Fig. 8(b). Fig. 8(c) is the simulated output of underdamped system with fixed and RFT discount factor. For both figures, it is obvious that if discount factor of the thread is fixed, it takes a lot of runs for the output to converge to the steady state, while for the thread with

---

**TABLE III**

**PERFORMANCE OF LOW-RUNNING PRODUCT (PRODUCT 1) IN CYCLES 0–3 WHEN THE CYCLE HAS VARIABLE LENGTH**

<table>
<thead>
<tr>
<th>Output without oscillation</th>
<th>Cycle 0</th>
<th>The The The</th>
<th>Cycle 1</th>
<th>The The The</th>
<th>Cycle 2</th>
<th>The The The</th>
<th>Cycle 3</th>
<th>The The The</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>decrease</td>
<td></td>
<td>decrease</td>
<td></td>
<td>decrease</td>
<td></td>
<td>decrease</td>
</tr>
<tr>
<td>fixed-MSE1</td>
<td>1.0833</td>
<td>48.1915</td>
<td>295.7041</td>
<td>89.2538</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CR-MSE1</td>
<td>0.4760</td>
<td>56.1%</td>
<td>24.0652</td>
<td>50.1%</td>
<td>151.0855</td>
<td>48.9%</td>
<td>41.9911</td>
<td>53.0%</td>
</tr>
<tr>
<td>fixed-variance1</td>
<td>0.2983</td>
<td>30.3193</td>
<td>148.4218</td>
<td>64.7702</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CR-variance1</td>
<td>0.2473</td>
<td>17.1%</td>
<td>22.1789</td>
<td>26.8%</td>
<td>139.9334</td>
<td>5.7%</td>
<td>39.6777</td>
<td>38.7%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Output with oscillation</th>
<th>Cycle 0</th>
<th>The The The</th>
<th>Cycle 1</th>
<th>The The The</th>
<th>Cycle 2</th>
<th>The The The</th>
<th>Cycle 3</th>
<th>The The The</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>decrease</td>
<td></td>
<td>decrease</td>
<td></td>
<td>decrease</td>
<td></td>
<td>decrease</td>
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<tr>
<td>fixed-MSE1</td>
<td>0.8630</td>
<td>58.1021</td>
<td>203.6856</td>
<td>79.88809</td>
<td></td>
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<tr>
<td>CR-MSE1</td>
<td>0.4566</td>
<td>47.1%</td>
<td>31.5682</td>
<td>45.7%</td>
<td>111.1545</td>
<td>45.4%</td>
<td>40.99763</td>
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<td>114.432</td>
<td>48.5%</td>
<td>41.55107</td>
<td>50.1%</td>
</tr>
</tbody>
</table>

---

**Fig. 7.** Output of products 1 and 2 in cycles 0–3 with fixed and CR discount factor when low-running product is manufactured with variable cycle lengths.
RFT discount factor, the output converge to the steady state much faster.

Our current approach does not consider metrology delay yet. In fact, because of the metrology delay, the fault-tolerant algorithm will be delayed several runs and some lots will out of spec, but many more lots will out of spec if the fault-tolerant approach is not used. So the RTF approach can still be used when there is metrology delay, though its effect is not as good as that without delay. If the metrology delay appeared after the step fault, then Fig. 9 is the simulated results with 1 run’s delay. Simulated outputs of overdamped and underdamped systems with fixed and RFT discount factor approaches are given in Fig. 9(a) and (b), respectively. From the figure, one can notice that by using RFT discount factor, the output of product 1 shows smaller deviations from the 252nd run on compared with that by using fixed discount factor. Also, product 2 by using RFT discount factor outperforms that by using fixed discount factor in the whole production cycles. Although this metrology delay prevents manufacturers from identifying fault by the output of the system in time, the proposed RFT approach still works.

VI. CONCLUSION

In this paper, the process with IMA(1,1) disturbance, deterministic linear drift and a step fault is studied. With the threaded-EWMA controller, the process outputs at each run in each cycle are derived. Based on system performance analysis, stability conditions and the causes of large deviations in the first few runs of each cycle are given. Cycled resetting (CR) algorithm for discount factor is proposed to deal with the large deviations at the beginning runs of each cycle as well as to achieve the minimum variance control. When a step fault appears, the system performance is also analyzed, and discount factor resetting fault-tolerant (RFT) approach is proposed. Simulation study shows that the approaches are very effective.

Although Theorems 2 and 3 only discussed the requirements of $g_n \left( \sum_{c=0}^{n-1} \xi_c + n \right)$, the performance of the system by using the proposed approach is superior compared with the original method (with fixed discount factor). Further research will focus on how to choose optimal $g_n \left( \sum_{c=0}^{n-1} \xi_c + n \right)$ to achieve the optimal control. In addition, the process with metrology delay will also be considered since it is a case of practical relevance.

APPENDIX A

Combine (1)–(5), and set $t = 0$, then

$$Y_n = \varphi_1 Y_{n-1} + \xi_1 \lambda_1 T_1 + \eta_n - \eta_{n-1}$$

$$= \varphi^2 Y_{n-2} + \sum_{k=0}^{2-1} \varphi^k \xi_1 \lambda_1 T_1 + \sum_{k=0}^{2-1} \varphi^k (\eta_{n-k} - \eta_{n-k-1})$$
Combine (1)–(5), then

\[ Y_{i_{0}+1} = \varphi_{1} Y_{i_{1},o} + \xi_{1} \lambda_{1} T_{1} + \eta_{i_{0}+1} - \eta_{j_{1},o} \]

\[ + \varphi_{1} \left[ \varphi_{1}^{j_{1},o-1} (\alpha_{1} + \xi_{1} T_{1} - \xi_{1} \bar{a}_{1} + \eta_{1}) + \sum_{k=0}^{j_{1},o-2} \varphi_{1}^{k} \xi_{1} \lambda_{1} T_{1} \right] \]

\[ + \sum_{k=0}^{j_{1},o-2} \varphi_{1}^{k+1} (\eta_{i_{1},o-k} - \eta_{j_{1},o-k-1}) \]

\[ = \varphi_{1}^{j_{1}+1} (\alpha_{1} + \xi_{1} T_{1} - \xi_{1} \bar{a}_{1} + \eta_{1}) + \sum_{k=0}^{j_{1},o-2} \varphi_{1}^{k+1} \left[ (\eta_{i_{1},o-k} - \eta_{j_{1},o-k-1}) + \xi_{1} \lambda_{1} T_{1} + \eta_{i_{0}+1} - \eta_{j_{1},o} \right] \]

\[ \text{for } t = 1, n = 1 \]

\[ Y_{i_{0}+n} = \varphi_{1} Y_{i_{n}+n-1} + \xi_{1} \lambda_{1} T_{1} + \eta_{i_{0}+n-1} - \eta_{j_{1},o} \]

\[ = \varphi_{1}^{n} Y_{i_{n}+n-2} + \sum_{k=0}^{n-2} \varphi_{1}^{k} \xi_{1} \lambda_{1} T_{1} \]

\[ + \sum_{k=0}^{n-2} \varphi_{1}^{k+1} (\eta_{i_{n}+n-k} - \eta_{j_{1},o+k-1}) \]

\[ = \varphi_{1}^{n-1} Y_{i_{0}+1} + \sum_{k=0}^{n-2} \varphi_{1}^{k} \xi_{1} \lambda_{1} T_{1} \]

\[ + \sum_{k=0}^{n-2} \varphi_{1}^{k+1} (\eta_{i_{0}+n-k} - \eta_{j_{1},o+n-k-1}) \]  \hspace{1cm} (B.1)

for \( t = 1 \) and \( n \geq 2 \); see equation (B.3) at the bottom of the next page, for \( t \geq 2 \) and \( n = 1 \)

\[ Y_{i_{0}+n} = \varphi_{1} Y_{i_{n}+n-1} + \xi_{1} \lambda_{1} T_{1} + \eta_{i_{0}+n-1} - \eta_{j_{1},o} \]

\[ = \varphi_{1}^{n} Y_{i_{n}+n-2} + \sum_{k=0}^{n-2} \varphi_{1}^{k} \xi_{1} \lambda_{1} T_{1} \]

\[ + \sum_{k=0}^{n-2} \varphi_{1}^{k+1} (\eta_{i_{n}+n-k} - \eta_{j_{1},o+k-1}) \]

\[ = \varphi_{1}^{n-1} Y_{i_{0}+1} + \sum_{k=0}^{n-2} \varphi_{1}^{k} \xi_{1} \lambda_{1} T_{1} \]

\[ + \sum_{k=0}^{n-2} \varphi_{1}^{k+1} (\eta_{i_{0}+n-k} - \eta_{j_{1},o+n-k-1}) \]  \hspace{1cm} (B.2)

for \( t = 1 \) and \( n \geq 2 \); see equation (B.3) at the bottom of the next page, for \( t \geq 2 \) and \( n = 1 \)

\[ Y_{i_{0}+n} = \varphi_{1} Y_{i_{n}+n-1} + \xi_{1} \lambda_{1} T_{1} + \eta_{i_{0}+n-1} - \eta_{j_{1},o} \]

\[ = \varphi_{1}^{n} Y_{i_{n}+n-2} + \sum_{k=0}^{n-2} \varphi_{1}^{k} \xi_{1} \lambda_{1} T_{1} \]

\[ + \sum_{k=0}^{n-2} \varphi_{1}^{k+1} (\eta_{i_{n}+n-k} - \eta_{j_{1},o+k-1}) \]

\[ = \varphi_{1}^{n-1} Y_{i_{0}+1} + \sum_{k=0}^{n-2} \varphi_{1}^{k} \xi_{1} \lambda_{1} T_{1} \]

\[ + \sum_{k=0}^{n-2} \varphi_{1}^{k+1} (\eta_{i_{0}+n-k} - \eta_{j_{1},o+n-k-1}) \]  \hspace{1cm} (B.4)

for \( t \geq 2 \) and \( n \geq 2 \).

Combining (B.1)–(B.4) and substituting (2) into them, Lemma 2 can be obtained.

\[ \text{APPENDIX C} \]

From (6), then

\[ \text{Var}(Y_{n}) = E(Y_{n} - EY_{n})^{2} \]

\[ = E \left[ \varphi_{1}^{n-1} \xi_{1} + \sum_{k=0}^{n-2} \varphi_{1}^{k} (\xi_{n-k} - \theta \xi_{n-k-1}) \right]^{2} \]

\[ = (\varphi_{1} - \theta)^{2} \frac{n}{\varphi_{1}^{2} \sigma^{2}}. \]  \hspace{1cm} (C.1)

Take variance for \( Y_{i_{0}} \sum_{c=0}^{n-1} i_{c} + i_{n} \), then see equation (C.2).

Take variance for \( Y_{i_{0}} \sum_{c=0}^{n-1} i_{c} + i_{n} \), and combine (C.2), then

\[ \text{Var} \left( \sum_{c=0}^{n-1} i_{c} + i_{n} \right) \]

\[ = E \left( \sum_{c=0}^{n-1} i_{c} + i_{n} - E \sum_{c=0}^{n-1} i_{c} + i_{n} \right)^{2} \]

\[ = E \left[ \varphi_{1}^{n-1} \left( \sum_{c=0}^{n-1} i_{c} + i_{n} - E \sum_{c=0}^{n-1} i_{c} + i_{n} \right) \right. \]

\[ + \sum_{k=0}^{n-2} \varphi_{1}^{k} \left( \xi_{n-k} - \theta \xi_{n-k-1} \right) \left( \sum_{c=0}^{n-1} i_{c} + i_{n} - E \sum_{c=0}^{n-1} i_{c} + i_{n} \right) \right]^{2} \]

\[ = \varphi_{1}^{n-1} \text{Var} \left( \sum_{c=0}^{n-1} i_{c} + i_{n} \right) \]

\[ + E \left[ \sum_{k=0}^{n-2} \varphi_{1}^{k} \left( \xi_{n-k} - \theta \xi_{n-k-1} \right) \left( \sum_{c=0}^{n-1} i_{c} + i_{n} - E \sum_{c=0}^{n-1} i_{c} + i_{n} \right) \right] \]

\[ = \varphi_{1}^{n-1} \text{Var} \left( \sum_{c=0}^{n-1} i_{c} + i_{n} \right) \]

\[ + E \left[ \sum_{k=0}^{n-2} \varphi_{1}^{k} \left( \xi_{n-k} - \theta \xi_{n-k-1} \right) \left( \sum_{c=0}^{n-1} i_{c} + i_{n} - E \sum_{c=0}^{n-1} i_{c} + i_{n} \right) \right] \]
\[
\sum_{c=0}^{\eta_2-1} \phi_{l,2}^{c+1} + \xi_1 \lambda_1 T_1 + \eta_{2} \sum_{c=0}^{\eta_2-1} \phi_{l,2}^{c+1} + \sum_{k=0}^{\eta_2-1} (\eta_{2} \sum_{c=0}^{\eta_2-1} \phi_{l,2}^{c+1} - \eta_{2} \sum_{c=0}^{\eta_2-1} \phi_{l,2}^{c+1} - \eta_{2} \sum_{c=0}^{\eta_2-1} \phi_{l,2}^{c+1})
\]

\[
= \phi_{l,2}^{\eta_2} \sum_{c=0}^{\eta_2-1} \phi_{l,2}^{c+1} + \xi_1 \lambda_1 T_1 + \eta_{2} \sum_{c=0}^{\eta_2-1} \phi_{l,2}^{c+1} + \sum_{k=0}^{\eta_2-1} (\eta_{2} \sum_{c=0}^{\eta_2-1} \phi_{l,2}^{c+1} - \eta_{2} \sum_{c=0}^{\eta_2-1} \phi_{l,2}^{c+1} - \eta_{2} \sum_{c=0}^{\eta_2-1} \phi_{l,2}^{c+1})
\]
Taking limit for (C.1) and (C.2), Lemma 3 can be got.

**APPENDIX D**

For cycle 0, minimum asymptotic variance is achieved from (8) when \( \varphi_1 = \theta \), i.e., optimal discount factor \( \lambda_1^* = (1 - \theta) / \xi_1 \); for cycle \( t (t \geq 1) \), after calculating the first and second order derivative of (9) with respect to \( \varphi_1 \), one can easily obtained the optimal \( \varphi_1^* = \theta \), i.e., optimal discount factor \( \lambda_1^* = (1 - \theta) / \xi_1 \) in order to obtain minimum asymptotic variance.

**APPENDIX E**

If a stable process behaves overdamped, one can conclude from Theorem 1 together with (10) and (11) that \( 0 < \varphi_1 < 1 \) and a small \( \varphi_1 \) is expected to accelerate the convergence rate, i.e., a large \( \lambda_1 \left( \sum_{c=0}^{n-1} i_c + n \right) \) is desired to reduce the large deviations at the beginning runs of each cycle. On the other hand, for the process with oscillation, one can also conclude from Theorem 1 that \( -1 < \varphi_1 < 0 \), i.e., \( 0 < |\varphi_1| = \xi_1 \lambda_1 \left( \sum_{c=0}^{n-1} i_c + n \right) < 1 \); in this situation, from (10) and (11) it is easy to know that \( \lambda_1 \left( \sum_{c=0}^{n-1} i_c + n \right) \) is required to be small to accelerate the convergence rate. With the increase of \( n \), the large deviation will disappear. At this time, minimum asymptotic variance is desired, i.e., the optimal discount factor is required. If \( \lambda_1 \left( \sum_{c=0}^{n-1} i_c + n \right) \) is denoted by linear combination of \( \lambda_1^* \) (see Lemma 5) and \( g_1 \left( \sum_{c=0}^{n-1} i_c + n \right) \), then Theorem 2 can be obtained.

**APPENDIX F**

Combine (1)–(5), (15) and (16), let \( t = 0 \), then

\[
Y_n = \varphi_1 Y_{n-1} + \xi_1 \lambda_1 T_1 + F_n - F_{n-1} \\
= \varphi_1^{n-1} Y_1 + \sum_{k=0}^{n-2} \varphi_1^k \xi_1 \lambda_1 T_1 + \sum_{k=0}^{n-2} \varphi_1^k (F_{n-k} - F_{n-k-1}) \\
= (1 - \varphi_1^{n-1}) T_1 + \varphi_1^{n-1} (\alpha_1 + \xi_1 T_1 - \xi_1 \alpha_1 + \varepsilon_1 + f_1 + \delta) \\
+ \gamma_2 \sum_{k=0}^{n-2} \varphi_1^k (f_{n-k} - f_{n-k-1} + \varepsilon_{n-k} - \theta \varepsilon_{n-k-1} + \delta).
\]

Taking mathematic expectation for \( Y_n \) in above equation, Lemma 6 can be got.

**APPENDIX G**

Combine (1)–(5), (15) and (16), then

\[
Y_{i_0+1} = \varphi_1 Y_{i_0} + \xi_1 \lambda_1 T_1 + F_{i_0+1} - F_{i_10} \\
= \varphi_1^{i_0} (\alpha_1 + \xi_1 T_1 - \xi_1 \alpha_1 + F_1) + (1 - \varphi_1^{i_0}) T_1 \\
+ \sum_{k=0}^{i_0+1} \varphi_1^k (F_{i_0-k} - F_{i_0-k-1} + F_{i_0+1} - F_{i_10}) \\
(\text{G.1})
\]

for \( t = 1 \) and \( n = 1 \);

\[
Y_{i_0+n} = \varphi_1 Y_{i_0+n-1} + \xi_1 \lambda_1 T_1 + F_{i_0+n} - F_{i_0+n-1} \\
= (1 - \varphi_1^{i_0+n-1}) T_1 + \varphi_1^{i_0+n-1} Y_{i_0+n-1} \\
+ \sum_{k=0}^{n-2} \varphi_1^k (F_{i_0+n-k} - F_{i_0+n-k-1}) \\
(\text{G.2})
\]
\[ Y_{t+1} = \varphi_1 Y_{t} + \xi_1 T_1 + F_{t+1} - F_{t} + \frac{\ell - 1}{m} \left( \sum_{i=0}^{m-1} \varphi_{t+i} \xi_1 T_1 \right) + \sum_{k=0}^{n-2} \varphi_{t+i}^k \left( \sum_{i=0}^{m-1} \varphi_{t+i} \right) \]

for \( t = 1 \) and \( n \geq 2 \); see equation (G.3) at the top of the page.

for \( t \geq 2 \) and \( n = 1 \)

\[ Y_{t+1} = \varphi_1 Y_{t} + \xi_1 T_1 + F_{t+1} - F_{t} + \frac{\ell - 1}{m} \left( \sum_{i=0}^{m-1} \varphi_{t+i} \xi_1 T_1 \right) + \sum_{k=0}^{n-2} \varphi_{t+i}^k \left( \sum_{i=0}^{m-1} \varphi_{t+i} \right) \]

for \( t \geq 2 \) and \( n \geq 2 \).

Combining (G.1)–(G.4), one can get equation (G.5) at the top of the page. Lemma 7 can be obtained by taking mathematic expectation for \( Y_{t+1} \) in (G.5).

\[ Y_{t+1} = \varphi_1 Y_{t} + \xi_1 T_1 + F_{t+1} - F_{t} + \frac{\ell - 1}{m} \left( \sum_{i=0}^{m-1} \varphi_{t+i} \xi_1 T_1 \right) + \sum_{k=0}^{n-2} \varphi_{t+i}^k \left( \sum_{i=0}^{m-1} \varphi_{t+i} \right) \]

References


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