## Homework 1: Answer Key

CS 311: Discrete Math for CS (Bulko)

Due 09/15/17 @ 11:59 pm

### Problem 1

(8 points each). Draw the truth table for the following compound propositions:

a. \((\neg p \lor q) \oplus \neg q\)

<table>
<thead>
<tr>
<th>(p)</th>
<th>(q)</th>
<th>(\neg p)</th>
<th>(\neg p \lor q)</th>
<th>(\neg q)</th>
<th>((\neg p \lor q) \oplus \neg q)</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>F</td>
<td>T</td>
<td>F</td>
<td>T</td>
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<td>T</td>
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<td>F</td>
<td>F</td>
<td>T</td>
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<td>F</td>
<td>T</td>
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<td>F</td>
<td>F</td>
<td>T</td>
<td>T</td>
<td>F</td>
<td>f</td>
</tr>
</tbody>
</table>

b. \((p \land \neg q) \lor (\neg p \land q)\)

<table>
<thead>
<tr>
<th>(p)</th>
<th>(q)</th>
<th>(\neg q)</th>
<th>(p \land \neg q)</th>
<th>(\neg p)</th>
<th>(\neg p \land q)</th>
<th>((p \land \neg q) \lor (\neg p \land q))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
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<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

c. \(((p \iff q) \land (q \iff r) \land (r \iff p))\)

<table>
<thead>
<tr>
<th>(p)</th>
<th>(q)</th>
<th>(r)</th>
<th>((p \iff q))</th>
<th>((q \iff r))</th>
<th>((r \iff p))</th>
<th>(((p \iff q) \land (q \iff r) \land (r \iff p)))</th>
</tr>
</thead>
<tbody>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
<td>T</td>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>
Problem 2

6) points). Write the following English sentence as a proposition:

a. “I will be on time to class if I get my stuff together tonight and do not sleep past my alarm.”

\[ p = \text{I will be on time to class} \]
\[ q = \text{I will get my stuff together tonight} \]
\[ r = \text{I do not sleep past my alarm} \]
\[ (q \land r) \Rightarrow p \]

b. “Nancy is doing her math homework, but Karen is not doing hers.”

\[ n = \text{Nancy is doing math} \]
\[ k = \text{Karen is doing math} \]
\[ (n \land \neg k) \]

Problem 3

(6 points each). Evaluate the following bit string expressions.
a. \(0011 \lor (1111 \land 0101)\)
\[
0011 \lor (1111 \land 0101)
\]
\[
= 0011 \lor (0101)
\]
\[
= 0111
\]

b. \(110010 \land (111000 \oplus 010110)\)
\[
110010 \land (111000 \oplus 010110)
\]
\[
= 110010 \land 101110
\]
\[
= 100010
\]

**Problem 4**

(6 points each). Determine if the following propositions are logically equivalent.

a. \(((p \lor q) \rightarrow r) \land ((p \rightarrow r) \land (q \rightarrow r))\) YES

Truth table, or:
\[
((p \lor q) \rightarrow r) \quad \text{Given}
\]
\[
\equiv \neg (p \lor q) \lor r \quad \text{Implication}
\]
\[
\equiv (\neg p \land \neg q) \lor r \quad \text{DeMorgan's Law}
\]
\[
\equiv (\neg p \lor r) \land (\neg q \lor r) \quad \text{Distributive}
\]
\[
\equiv (p \rightarrow r) \lor (q \rightarrow r) \quad \text{Implication x2}
\]

b. \((p \rightarrow (q \land r)) \land (p \rightarrow q) \land (p \rightarrow r)\) YES

Truth table, or:
\[
(p \rightarrow (q \land r)) \quad \text{Given}
\]
\[
\equiv \neg p \lor (q \land r) \quad \text{Implication}
\]
\[
\equiv (\neg p \lor q) \land (\neg p \land r) \quad \text{Distributive}
\]
\[
\equiv (p \rightarrow q) \land (p \rightarrow r) \quad \text{Implication x2}
\]

c. \((p \rightarrow (q \lor r)) \land (p \land \neg q) \rightarrow r\) YES

Truth table, or:
\[
(p \rightarrow (q \lor r)) \quad \text{Given}
\]
\[
\equiv \neg p \lor (q \lor r) \quad \text{Implication}
\]
\[
\equiv (\neg p \lor q) \lor r \quad \text{Associative}
\]
\[
\equiv (\neg (p \land \neg q) \lor (r) \quad \text{DeMorgan's Law}
\]
\[
\equiv (p \land \neg q) \rightarrow r \quad \text{Implication}
\]
Problem 5

(5 points each). Determine if each proposition is satisfiable.

a. \((p \land q \land r) \lor p \rightarrow r\)  Satisfiable. Ex: \(p = T, q = T, r = T\) means \(T \rightarrow T\) which is \(T\).

b. \((\neg q \oplus r) \land p \lor \neg q\)  Satisfiable. The \(\land\) is done before the \(\lor\), so as long as \(q\) is \(F\), you end up with \((\text{something}) \lor T\), which is \(T\).

Problem 6

(12 points each statement; 2 per partial answer). Write the converse, inverse and contrapositive of each statement.

a. “If Sandra finishes her work, she will go to the basketball game unless it snows.”

Converse: If Sandra has gone to the basketball game, then she has finished her work and it is not snowing.

Inverse: If Sandra has not finished her work or it is snowing, then Sandra will not go to the basketball game.

Contrapositive: If Sandra doesn’t go to the basketball game, then she didn’t finish her work or it is snowing.

b. “If \(n\) is an even integer, then \(n^2\) is an even integer.”

Converse: If \(n^2\) is an even integer, then \(n\) is an even integer.

Inverse: If \(n\) is not an even integer, then \(n^2\) is not an even integer.

Contrapositive: If \(n^2\) is not an even integer, then \(n\) is not an even integer.