Homework 3 ANSWERS
CS 311: Discrete Math for CS (Bulko)
Due 09/29/17 @ 11:59 pm

Problem 1

(2 points each). Rewrite each statement using quantifiers. Let D = {everything}, the set of all things.
Q(x): is male
C(x): is a cat
R(x): calico coloring
F(x): is fluffy
M(x): eat mice
B(x): can bark

a. No male cats are calico colored.
\[ \forall x((C(x) \land Q(x)) \rightarrow \neg R(x)) \]

b. All cats eat mice.
\[ \forall x(C(x) \rightarrow M(x)) \]

c. There is a calico cat that barks.
\[ \exists x(C(x) \land R(x) \land B(x)) \]

d. Cats are fluffy.
\[ \forall x(C(x) \rightarrow F(x)) \]

Problem 2

(12 points). Write each statement twice, once using the quantifier \( \forall \) and once using the quantifier \( \exists \). Let D = {everything}, the set of all things.

S(x): stays up late
P(x): is a programmer
K(x): Computer science student
C(x): College
J(x): has had coffee

a. It is not the case that every programmer in college stays up late.
\[\neg \forall x [(C(x) \land P(x)) \to S(x)] \text{ rewrite as } \exists x [\neg (C(x) \land P(x)) \to S(x)]\]

b. There is a student in computer science who has not had coffee.
\[\exists x (K(x) \land \neg J(x)) \text{ rewrite as } \neg \forall x (K(x) \to J(x))\]

**Problem 3**

For each problem below, prove the conclusion follows logically from the premises using Rules of Inference. Be sure to show every step, and clearly identify the rationale for each step. (Tip: the left menu of the class web page contains a link to a complete list of Rules of Inference.)

**a. (10 points)**

Premises:
\[(p \land q) \to r\]
\[r \to s\]
\[\neg s\]

Conclusion to show:
\[\neg p \lor \neg q\]

(1) \(\neg s\) GIVEN
(2) \(r \to s\) GIVEN
(3) \(\neg r\) MODUS TOLLENS 1, 2
(4) \((p \land q) \to r\) GIVEN
(5) \(\neg(p \land q)\) MODUS TOLLENS 3, 4
(6) \(\neg p \lor \neg q\) DeMORGAN’S LAW

**b. (10 points)**

Premises:
\[p \to q\]
\[r \to s\]
\[(q \lor s) \to t\]
\[\neg t\]

Conclusion to show:
\[\neg p \land \neg r\]

(1) \(\neg t\) GIVEN
(2) \((q \lor s) \to t\) GIVEN
(3) \(\neg(q \lor s)\) MODUS TOLLENS 1, 2
(4) \(\neg q \land \neg s\) DeMORGAN’S LAW
(5) \(\neg q\) SIMPLIFICATION 4
(6) \(p \to q\) GIVEN
(7) \(\neg p\) MODUS TOLLENS 5, 6
(8) \(\neg s\) SIMPLIFICATION 4
c. (10 points)
Premises:
\[ p \rightarrow q \]
\[ r \rightarrow s \]
\[ (s \land q) \rightarrow t \]
\[ \neg t \]

Conclusion to show:
\[ \neg r \lor \neg p \]

(1) \( (s \land q) \rightarrow t \) GIVEN
(2) \( \neg t \) GIVEN
(3) \( \neg (s \land q) \) MODUS TOLLENS 1, 2
(4) \( \neg s \lor \neg q \) DeMORGAN’S LAW
(5) \( p \rightarrow q \) GIVEN
(6) \( r \rightarrow s \) GIVEN
(7) \( \neg p \) MODUS TOLLENS 4, 5
(8) \( \neg r \) MODUS TOLLENS 4, 6
(7)\( \neg r \lor \neg p \) (5)(6)(4) AND THE RULE OF (disjunctive) MODUS TOLLENS

Problem 4

(25 points total). Determine the truth value of the following statements, where the domain is all integers.
Let \( p(x) \), \( q(x) \) and \( r(x) \) be the following:
\[ p(x): x^2 - 7x + 10 = 0 \]
\[ q(x): x^2 - 2x - 3 = 0 \]
\[ r(x): x < 0 \]

a) \( \forall x[q(x) \rightarrow r(x)] \) FALSE

b) \( \exists x[q(x) \rightarrow r(x)] \) TRUE

c) \( \exists x[p(x) \rightarrow r(x)] \) TRUE
Problem 5

(25 points) Give the reasons for the steps verifying the following *Tip: refer to the left side of the class web page for a complete list of inference rules.*

Premises:

\[-p \lor q \rightarrow r\]
\[r \rightarrow (s \lor t)\]
\[\neg s \land \neg u\]
\[\neg u \rightarrow \neg t\]

Conclusion to show:

\[p\]

Steps:

1. \[\neg s \land \neg u\] GIVEN
2. \[\neg u\] SIMPLIFICATION 1
3. \[\neg u \rightarrow \neg t\] GIVEN
4. \[\neg t\] MODUS PONENS 2, 3
5. \[\neg s\] SIMPLIFICATION 1
6. \[\neg s \land \neg t\] CONJUNCTION 4, 5
7. \[r \rightarrow (s \lor t)\] GIVEN
8. \[\neg (s \lor t) \rightarrow \neg r\] CONTRAPOSITIVE 7
9. \[\neg s \land \neg t\] \[\rightarrow \neg r\] DeMORGAN’S LAW
10. \[\neg r\] MODUS PONENS 6, 9
11. \[\neg p \lor q\] \[\rightarrow r\] GIVEN
12. \[\neg r\] \[\rightarrow \neg (\neg p \lor q)\] CONTRAPOSITIVE 11
13. \[\neg r\] \[\rightarrow (p \land \neg q)\] DeMORGAN’S LAW AND LAW OF DOUBLE NEGATION
14. \[p \land \neg q\] MODUS PONENS 10, 13
15. \[p\] SIMPLIFICATION 14