Problem 1

(5 points each) In the following statements the domain is the set of all nonzero integers. Determine the truth value of each statement.

(a) \( \exists x \exists y [xy = 1] \)

(b) \( \exists x \forall y [xy = 1] \)

(c) \( \forall x \exists y [xy = 1] \)

Problem 2

(20 points) Prove the conclusion below by using a Proof by Contradiction.

Premises:
- \( p \rightarrow q \)
- \( \neg r \lor s \)
- \( p \lor r \)

Conclusion:
- \( \neg q \rightarrow s \)

Problem 3

(20 points) Give a direct proof for the following.

Premises:
- \( \neg p \leftrightarrow q \)
- \( q \rightarrow r \)
- \( \neg r \)

Conclusion to show:
- \( p \)
Problem 4

(15 points) Prove the conclusion follows logically from the premises using Rules of Inference. Be sure to show every step.

Premises:

- $p \rightarrow q$
- $q \rightarrow (r \land s)$
- $\neg r \lor (\neg t \lor u)$
- $p \land t$

Conclusion to show:

- $u$

Problem 5

(30 points) Use an informal proof to prove that $\sqrt[3]{2}$ is irrational.

Tip 1: Use a Proof by Contradiction.

Tip 2: First prove that if $k^3$ is even, then $k$ is even. Then use this theorem in your proof.*