Problem 1

(15 points). Let $g : \mathbb{N} \to \mathbb{N}$ be defined by $g(n) = 2n$. If $A = \{1, 2, 3, 4\}$ and $f : A \to \mathbb{N}$ is given by $f = \{(1, 2), (2, 3), (3, 5), (4, 7)\}$, find $g \circ f$.

Problem 2

(25 points). Given predicates $p(x), q(x)$ in an arbitrary universe $U$, prove $\exists x [p(x) \lor q(x)] \iff (\exists x p(x)) \lor (\exists x q(x))$.

(Hint: using "without loss of generality" may make this proof a little shorter.)

Problem 3

(25 points) Prove that $f : A \to B$ is one-to-one if and only if $|f^{-1}(b)| \leq 1$ for all $b \in B$.

Problem 4

(5 points each) For each of the following functions $f : \mathbb{R} \to \mathbb{R}$, determine whether or not $f$ is invertible, and, if so, determine $f^{-1}$.

a) $f : \{(x, y) \mid ax + by = c, b \neq 0\}$

b) $f : \{(x, y) \mid y = x^3\}$

c) $f : \{(x, y) \mid y = x^4 - x\}$
(20 points) A function \( f : \mathbb{R} \to \mathbb{R} \) is said to be strictly *increasing* if for the real numbers \( x, y \) we have \( x < y \Rightarrow f(x) < f(y) \). Prove that if \( f, g : \mathbb{R} \to \mathbb{R} \) are strictly increasing functions, then \( g \circ f : \mathbb{R} \to \mathbb{R} \) is strictly increasing.