Problem 1

(20 points) Use mathematical induction to show that $2^n > n^2 + n$, when $n$ is an integer and $n \geq 5$.

Problem 2

(20 points) Use mathematical induction to show that:

$$\frac{1}{1 \cdot 3} + \frac{1}{3 \cdot 5} + \ldots + \frac{1}{(2n-1)(2n+1)} = \frac{n}{2n+1}$$

whenever $n$ is a positive integer.

Problem 3

(20 points) Verify that the program segment:

```python
if x > y then
    max := x
else
    max := y
```

is correct with the initial assertion $T$ and the final assertion $(x > y \land max = x) \lor (x \leq y \land max = y)$

Hint: Use proof by cases.

Problem 4

(25 points) Given the following program segment, we claim that the code calculates $q$ and $r$, where $q$ is the quotient and $r$ is the remainder when $a$ divides $d$. Find a loop invariant and verify that it is partially correct with respect to the initial assertion $x = "a, d are positive integers"$ and the final assertion $y = "q, r are integers such that a = dq + r and 0 \leq r < d"$. (Remember that a loop invariant can be shown to be partially correct if it holds during initialization, maintenance, and termination.)

```python
r := a
q := 0
while r \geq d
    r := r - d
    q := q + 1
```
Problem 5

(15 points) Verify that the program segment:

\[
\begin{align*}
x &:= 4 \\
z &:= x - y \\
\text{if } y > 1 \text{ then} \\
&\quad z := z + 3 \\
\text{else} \\
&\quad z := 0
\end{align*}
\]

is correct given the initial assertion \( y = 3 \) and the final assertion \( z = 4 \).