Rules of Inference

Modus Ponens
\[ p \\ p \rightarrow q \]
\[ \therefore q \]
\[ (p \land (p \rightarrow q)) \rightarrow q \]

Modus Tollens
\[ \neg q \\ p \rightarrow q \]
\[ \therefore \neg p \]
\[ (\neg q \land (p \rightarrow q)) \rightarrow \neg p \]

Disjunctive Syllogism
\[ p \lor q \\ \neg p \]
\[ \therefore q \]
\[ ((p \lor q) \land \neg p) \rightarrow q \]

Hypothetical Syllogism
\[ p \rightarrow q \\ q \rightarrow r \]
\[ \therefore p \rightarrow r \]
\[ ((p \rightarrow q) \land (q \rightarrow r)) \rightarrow (p \rightarrow r) \]

Addition
\[ p \]
\[ \therefore p \lor q \]
\[ p \rightarrow (p \lor q) \]

Simplification
\[ p \land q \]
\[ \therefore p \]
\[ (p \land q) \rightarrow p \]

Conjunction
\[ p \\ q \]
\[ \therefore p \land q \]
\[ (p) \land (q) \rightarrow p \land q \]

Resolution
\[ p \lor q \\ \neg p \lor r \]
\[ \therefore q \lor r \]
\[ ((p \lor q) \land (\neg p \lor r)) \rightarrow q \lor r \]