Homework 3
CS 311: Discrete Math for CS (Bulko)
Due 09/28/18 @ 11:59 pm

Problem 1

(5 points) Convert the following predicate to one that has no negation in it. Justify each step.
\[ \exists x \neg (P(x) \to \exists y (\neg Q(x, y) \land \neg R(x, y))) \]

Problem 2

(5 points each) Express the following English statements with quantifiers and elementary predicates.

a. There exists a negative integer that is odd. Let the domain be the set of all integers.
b. If \( x \) is even, then \( x \) is not divisible by 5. Let the domain be the set of all integers.
c. Every freshman CS major who is not in CS312 is in CS314. Let the domain be the set of all UT freshmen.
d. There exists a student who is both a CS major and a Math major. Let the domain be the set of all UT students.

Problem 3

(5 points each) What rule of inference is used in each of these arguments? *Tip: refer to the left side of the class web page for a complete list of inference rules.*

a. Brenda is an excellent footballer. If Brenda is an excellent footballer, then she can be on the team. Therefore, Brenda can be on the team.
b. It is raining outside. It is sunny outside. It is raining and sunny outside.
c. It is either hotter than 100 degrees today or the pollution is dangerous. It is less than 100 degrees outside today. Therefore, the pollution is dangerous.
d. If I workout tonight, then I do not have to work out tomorrow. If I don't workout tomorrow, I need to workout the day after tomorrow. Therefore, if I workout tonight, then I need to workout the day after tomorrow.
e. If it snows today, the university will declare a holiday. The university did not declare a holiday today. Therefore, it did not snow today.
Problem 4

(5 points each). Let P(x), Q(x) and R(x) be the following statements.

\[ P(x) : x^2 - 8x + 15 = 0 \]
\[ Q(x) : x \text{ is odd} \]
\[ R(x) : x > 0 \]

Let the domain be the set of all integers.

Find the truth value of each of the following statements. If a statement is false, provide a counterexample.

a. \[ \exists x [Q(x) \rightarrow P(x)] \].
b. \[ \forall x [Q(x) \rightarrow P(x)] \].
c. \[ \exists x [R(x) \rightarrow P(x)] \].
d. \[ \forall x [(P(x) \lor Q(x)) \rightarrow R(x)] \].

Problem 5

(5 points each). For each of these collections of premises, what relevant conclusion or conclusions can be drawn? Explain the rules of inference used to obtain each conclusion from the premises. *Tip: refer to the left side of the class web page for a complete list of inference rules.*

a. "I am either clever or lucky", "I am not lucky", "If I am lucky, then I will win the lottery."
b. "If I play basketball, then I am sore the next day", "I use the hot tub if I am sore", "I did not use the hot tub".
c. "I am either dreaming or hallucinating", "I am not dreaming", "If I am hallucinating, I see birds swimming in the ocean."
d. "I love candy", "I love chocolates".

Problem 6

(10 points). Show that the hypotheses "If you send me an email message, then I will finish doing the job", "If you do not send me an email message, then I will go to bed early" and "If I go to bed early, then I will wake up feeling refreshed" leads to the conclusion "If I do not finish doing the job, then I will wake up feeling refreshed."