Big-O Notation
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**Definition:** Let $f$ and $g$ be functions from the set of integers or the set of real numbers to the set of real numbers. We say that $f(x)$ is $O(g(x))$ if there are constants $C$ and $k$ such that

$$|f(x)| \leq C|g(x)|$$

whenever $x > k$. This is read as “$f(x)$ is big-oh of $g(x)$” or “$f(x)$ is order $g(x)$”.

- The constants $C$ and $k$ in the definition above are called *witnesses* to the relationship “$f(x)$ is $O(g(x))$”.
- To establish that $f(x)$ is $O(g(x))$, we need to find one pair of constants $C$ and $k$ such that $|f(x)| \leq C|g(x)|$ whenever $x > k$. 
Big-O of Polynomials

**Theorem:** Let $f(x) = a_nx^n + a_{n-1}x^{n-1} + \ldots + a_1x + a_0$, where $a_0, a_1, \ldots, a_n \in \mathbb{R}$.

Then $f(x)$ is $O(x^n)$. 
Big-O of Combinations of Functions

**Theorem:** Suppose that $f_1(x)$ is $O(g_1(x))$ and $f_2(x)$ is $O(g_2(x))$. Then $(f_1 + f_2)(x) = O(\max(|g_1(x)|, |g_2(x)|))$.

**Corollary:** Suppose that $f_1(x)$ and $f_2(x)$ are both $O(g(x))$. Then $(f_1 + f_2)(x) = O(g(x))$.

**Theorem:** Suppose that $f_1(x)$ is $O(g_1(x))$ and $f_2(x)$ is $O(g_2(x))$. Then $(f_1f_2)(x) = O(g_1(x)g_2(x))$. 