Problem 1

(5 points each). Determine whether the given graph is connected.

a.

![Graph](image1)

b.

Problem 2

(10 points each). Determine whether each of these graphs is strongly connected and if not, whether it is weakly connected.

a.

![Graph](image2)
Problem 3

(10 points). Find all the cut vertices of the given graph.

Problem 4

(15 points each).

(a) A vertex is *pendant* if and only if it has degree one. Suppose that \( v \) is an endpoint of a cut edge. Prove that \( v \) is a cut vertex if and only if this vertex is not pendant.

(b) Prove that a graph with \( v \) vertices and \( e \) edges has at least \( v - e \) connected components.

(c) What is the maximum number of connected components that can result from removing the edges in a length-\( k \) cycle from a connected graph? Explain your answer.

(d) Given a graph \( G = (V, E) \), prove that if the degree of each vertex is at least \( \frac{|V| - 1}{2} \), then the graph
is connected.