Homework 3
CS 311: Discrete Math for CS (Bulko)
Due 02/16/18 @ 11:59 pm

Problem 1

(5 points). Convert the following predicate to one that has no negation in it.

\( \neg \forall x (P(x) \rightarrow \exists x (\neg Q(x, y) \land \neg R(x, y))) \)

Problem 2

(5 points each). What rule of inference is used in each of these arguments? *Tip: refer to the left side of the class web page for a complete list of inference rules.*

b. Brenda is an excellent footballer. If Brenda is an excellent footballer, then she can be on the team. Therefore, Brenda can be on the team.
c. If it rains today, the university will close. The university is not closed today. Therefore, it did not rain today.
d. It is either hotter than 100 degrees today or the pollution is dangerous. It is less than 100 degrees outside today. Therefore, the pollution is dangerous.
e. If I work all night on this homework, then I can answer all the exercises. If I answer all the exercises, I will understand the material. Therefore, if I work all night on this homework, then I will understand the material.

Problem 3

(5 points). Show that the hypotheses "If you send me an email message, then I will finish doing the job", "If you do not send me an email message, then I will go to bed early" and "If I go to bed early, then I will wake up feeling refreshed" leads to the conclusion "If I do not finish doing the job, then I will wake up feeling refreshed."
Problem 4

(5 points each). For each of these collections of premises, what relevant conclusion or conclusions can be drawn? Explain the rules of inference used to obtain each conclusion from the premises. *Tip: refer to the left side of the class web page for a complete list of inference rules.*

a. "I am either clever or lucky", "I am not lucky", "If I am lucky, then I will win the lottery."
b. "If I play basketball, then I am sore the next day", "I use the hot tub if I am sore", "I did not use the hot tub".
c. "I am either dreaming or hallucinating", "I am not dreaming", "If I am hallucinating, I see birds swimming in the ocean."

Problem 5

(10 points each). Prove the following statements. Clearly state the proof technique you use. Possible proof techniques include direct proof, indirect proof, proof by contradiction, and proof by cases.

a. If \(n^2\) is even, then so is \(n\).
b. If \(42^n - 1\) is prime, then \(n\) must be odd.
c. If \(r\) and \(x\) are both rational numbers, then \(r + x\) is also a rational number.
d. If \(r\) is a rational number and \(x\) is an irrational number, then \(r + x\) is irrational. (Hint: use the result from Problem 5c).
e. \(\sqrt{2}\) is irrational. (Hint 1: use proof by contradiction, Hint 2: use the result from Problem 5a, Hint 3: a rational number can always be written uniquely in "lowest terms", meaning that the numerator and the denominator have no common factors.)