Problem 1

(10 points each).

a) Prove that $n! > 3^n$ if $n$ is an integer greater than 6.
b) Find all non-negative integers $n$ such that $3^n > 4n!$.

Problem 2

(10 points). For every positive integer $n$, prove that $7^n - 3^n$ is divisible by 4.

Problem 3

(10 points each). Prove each of the following by mathematical induction:

a) $\sum_{i=1}^{n} i(2^i) = 2 + (n - 1)2^{n+1}$
b) $\sum_{i=1}^{n} 2(3^{i-1}) = 3^n - 1$
c) $\sum_{i=1}^{n} (i)(i!) = (n + 1)! - 1$
d) Any set of $n$ elements, where $n$ is a positive integer, has $2^n$ subsets.

Problem 4

(10 points each). Prove each of the following statements using proof by contradiction and the well-ordering principle.

a) Every positive integer $> 1$ has at least one prime divisor.
b) Every positive integer $> 1$ can be written as a product of 2 or more primes.
Problem 5

(10 points). Prove or disprove (give a counterexample) the following statement: if $a, b, c,$ and $d$ are integers such that $ab|cd,$ then $a|c$ or $a|d.$