Problem 1

(10 points each). Draw a graph with the specified properties or show that no such graph exists.

a. A graph with 4 vertices of degrees 1, 1, 2, and 3.
b. A graph with 4 vertices of degrees 1, 1, 3, and 3.
c. A simple graph with 4 vertices of degrees 1, 1, 3, and 3.
d. A simple graph contains 8 vertices with degrees 0, 1, 2, 3, 4, 5, 6, 7.
e. A simple graph contains 4 vertices and 12 edges.
f. A graph contains 4 vertices with degrees 1, 2, 2, 3.

Problem 2

(15 points each). Find the in-degree and the out-degree of each vertex in the graph shown in the picture.

Problem 3

(10 points). Prove or disprove the following: "in a simple undirected graph \( G \) with at least two vertices, it is possible that all vertices have different degrees."
Problem 4

(15 points) A simple graph is called 3-regular if every vertex has degree 3. Show that any 3-regular graph has an even number of vertices. (Hint: use the Handshaking Theorem).