### Problem 1

(10 points each)

a) Show the adjacency list for the graph below.

b) What are the in and out degrees of each vertex?

![Graph](image)

### Problem 2

(10 points each)

a) Draw an undirected graph that corresponds with the adjacency matrix below.

\[
M = \begin{bmatrix}
0 & 1 & 1 & 1 & 0 \\
1 & 0 & 1 & 0 & 0 \\
1 & 1 & 0 & 1 & 1 \\
1 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0
\end{bmatrix}
\]

b) Show the adjacency matrix for the graph below.

![Graph](image)
Problem 3

(10 points) Find the adjacency matrix of this graph.

Problem 4

(5 points).

Determine whether each of the following statements is true or false. Support your answer with a simple example.

(a) In a directed graph, the sum of the outdegrees equals the sum of the indegrees.

(b) A graph where every vertex has at least degree \(d\) requires at least \(d\) colors to color each vertex of the graph so that no two adjacent vertices are assigned the same color.

Problem 5

(10 points).

a) A bipartite graph can be described as a graph \(G = (V,E)\) where \(V = V_1 \cup V_2\) and \(E \subseteq V_1 \times V_2\). What is the sum of the degrees of the vertices in \(V_1\) in terms of \(|V_1|, |V_2|,\) and \(|E|\)?

b) For a simple undirected graph \(G = (V,E)\) with \(n\) vertices, where \(n\) is even, a perfect matching is a set \(S \subseteq E\) where \(|S| = n/2\), and every vertex of the graph is incident to exactly one of the edges in the set. If a graph has a perfect matching, does this imply that it is bipartite?
Problem 6

(10 points each)

Determine whether or not the graphs below are isomorphic. Explain your answer.

Problem 7

(10 points each)

Determine whether or not the graphs below are isomorphic. Explain your answer.