Big-O Notation
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**Definition:** Let $f$ and $g$ be functions from the set of integers or the set of real numbers to the set of real numbers. We say that $f(x)$ is $O(g(x))$ if there are constants $C$ and $k$ such that

$$|f(x)| \leq C|g(x)|$$

whenever $x > k$. This is read as “$f(x)$ is big-oh of $g(x)$” or “$f(x)$ is order $g(x)$”.

- The constants $C$ and $k$ in the definition above are called *witnesses* to the relationship “$f(x)$ is $O(g(x))$”.
- To establish that $f(x)$ is $O(g(x))$, we need to find *one* pair of constants $C$ and $k$ such that $|f(x)| \leq C|g(x)|$ whenever $x > k$. 
**Theorem:** Let \( f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0, \)
where \( a_0, a_1, \ldots, a_n \in \mathbb{R}. \)

Then \( f(x) \) is \( O(x^n). \)
Big-O of Combinations of Functions

**Theorem:** Suppose that \( f_1(x) \) is \( O(g_1(x)) \) and \( f_2(x) \) is \( O(g_2(x)) \). Then \( (f_1 + f_2)(x) = O( \max( |g_1(x)|, |g_2(x)| ) ) \).

**Corollary:** Suppose that \( f_1(x) \) and \( f_2(x) \) are both \( O(g(x)) \). Then \( (f_1 + f_2)(x) = O(g(x)) \).

**Theorem:** Suppose that \( f_1(x) \) is \( O(g_1(x)) \) and \( f_2(x) \) is \( O(g_2(x)) \). Then \( (f_1f_2)(x) = O( g_1(x)g_2(x) ) \).