Problem 1

(5 points). Convert the following predicate to one that has no negation in it. Justify each step.
\[ \neg \forall x((P(x) \lor S(x)) \rightarrow \exists y(\neg Q(x, y) \land \neg R(x, y))) \]

Problem 2

(10 points each). Express the following English statements with quantifiers and elementary predicates.
a. There exists a negative integer that is odd. Let the domain be the set of all integers.
b. If \( x \) is even, then \( x \) is not divisible by 5. Let the domain be the set of all integers.
c. There exists a student who is both a CS major and a Math major. Let the domain be the set of all UT students.
d. Every freshman CS major who is not in CS312 is in CS314. Let the domain be the set of all UT freshmen.

Problem 3

(5 points each). Write each statement twice, once using the quantifier \( \forall \) and once using the quantifier \( \exists \). Let \( D = \{ \text{everything} \} \), the set of all things.
a. It is not the case that every programmer in college stays up late.
b. There is a student in computer science who has not had coffee.

Problem 4

(5 points each). What rule of inference is used in each of these arguments? *Tip: refer to the left side of the class web page for a complete list of inference rules. *
a. Koalas live in New Zealand and eat eucalyptus leaves. Therefore, koalas eat eucalyptus leaves.
b. Brenda is an excellent football player. If Brenda is an excellent football player, then she can be on the team. Therefore, Brenda can be on the team.
c. If it rains today, the university will close. The university is not closed today. Therefore, it did not rain today.
d. It is either hotter than 100 degrees today or the pollution is dangerous. It is less than 100 degrees outside today. Therefore, the pollution is dangerous.
e. If I work all night on this homework, then I can answer all the exercises. If I answer all the exercises, I
will understand the material. Therefore, if I work all night on this homework, then I will understand the material.

Problem 5

(10 points). Use rules of inference to arrive to the conclusion.

If I get my Christmas bonus and my friends are free, I will take a road trip with my friends. If my friends don’t find a job after Christmas, then they will be free. I got my Christmas bonus and my friends did NOT find a job after Christmas. Therefore, I will take a road trip with my friends!

Problem 6

(10 points). Show that the hypotheses ”If you let me know the details, then I will finish doing the job”, ”If you do not let me know the details, then I will go to bed early” and ”If I go to bed early, then I will wake up feeling refreshed” leads to the conclusion ”If I do not finish doing the job, then I will wake up feeling refreshed.”