Foundations of Computer Security

Lecture 49: Public Key Encryption II

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The **Rivest-Shamir-Adelman (RSA)** algorithm relies on the difficulty of factoring large numbers.

Two keys, $e$ and $d$, are used for encryption and decryption. The algorithm is such that:

$$\{\{P\}_d\}_e = P = \{\{P\}_e\}_d.$$  

A plaintext block $P$ is encrypted as $(P^e \mod n)$. $d$ is chosen so that:

$$(P^e)^d \mod n = P.$$  

An interceptor would have to factor $P^e$ to recover the plaintext. The legitimate receiver knows $d$ and merely computes $(P^e)^d \mod n = P$, which is much easier.
A public key system can be based on any one-way function. A rich source is the set of NP-complete problems. These are infeasible to solve, but a solution can be checked in polynomial time.

Merkle and Hellman proposed a public key system based on the knapsack problem: given a set of integers and a target sum, find a subset of the integers that sum to the target.

The algorithm is theoretically very secure, but has practical weaknesses.
Authentication with Public Keys

Assume $K_a$ is A’s public key. Suppose B sends the following message to A: $\{M\}_{K_a}$. What assurances does A have?

1. No-one intercepting the message could read it. Why?
2. He can’t be sure it actually came from B. Why not?

Thus, encryption with the public key is a privacy transformation, but not an authenticity transformation.
Using RSA, B sends $\{M\}_{K_b^{-1}}$ to A. If A can decrypt it using $K_b$, what assurance is gained?

1. A is sure it originated with B. Why?
2. But, someone intercepting the message might read it. Why?

Thus, encryption with the private key is an *authenticity* transformation, not a *privacy* transformation.

*Note this only works in RSA, because:*

$$\{\{P\}_d\}_e = P = \{\{P\}_e\}_d.$$

In other public key systems, you typically need two pairs of keys: one pair for privacy and the other pair for “signing” (authenticity).
Lessons

- RSA is the most widely used public key cryptosystem.
- RSA is symmetric in the use of keys; most public key schemes are not.
- A public key encryption can be used for authenticity or for privacy but not both at once.

Next lecture: Cryptographic Hash Functions