Foundations of Computer Security

Lecture 52: Diffie-Hellman Key Exchange

Dr. Bill Young
Department of Computer Sciences
University of Texas at Austin

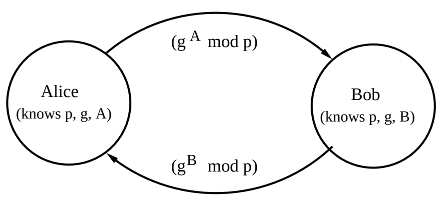
Diffie-Hellman Key Exchange

The question of key exchange was one of the first problems addressed by a cryptographic protocol. *This was prior to the invention of public key cryptography.*

The Diffie-Hellman key agreement protocol (1976) was the first practical method for establishing a shared secret over an unsecured communication channel.

The point is to agree on a key that two parties can use for a symmetric encryption, in such a way that an eavesdropper cannot obtain the key.

Diffie-Hellman Algorithm



Steps in the algorithm:

1. Alice and Bob agree on a prime number p and a base g.
2. Alice chooses a secret number a, and sends Bob $(g^a \mod p)$.
3. Bob chooses a secret number b, and sends Alice $(g^b \mod p)$.
4. Alice computes $((g^b \mod p)^a \mod p)$.
5. Bob computes $((g^a \mod p)^b \mod p)$.

Both Alice and Bob can use this number as their key. Notice that p and g need not be protected.

Diffie-Hellman Example

1. Alice and Bob agree on $p = 23$ and $g = 5$.
2. Alice chooses $a = 6$ and sends $5^6 \mod 23 = 8$.
3. Bob chooses $b = 15$ and sends $5^{15} \mod 23 = 19$.
4. Alice computes $19^6 \mod 23 = 2$.
5. Bob computes $8^{15} \mod 23 = 2$.

Then 2 is the shared secret.

Clearly, much larger values of a, b, and p are required. An eavesdropper cannot discover this value even if she knows p and g and can obtain each of the messages.

Diffie-Hellman Security

Suppose p is a prime of around 300 digits, and a and b at least 100 digits each.

Discovering the shared secret given g, p, $g^a \mod p$ and $g^b \mod p$ would take longer than the lifetime of the universe, using the best known algorithm. This is called the *discrete logarithm problem*.

Lessons

- How can two parties agree on a secret value when all of their messages might be overheard by an eavesdropper?
- The Diffie-Hellman algorithm accomplishes this, and is still widely used.
- With sufficiently large inputs, Diffie-Hellman is very secure.

Next lecture: Digital Signatures