CS429: Computer Organization and Architecture

Optimization II

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Cache Performance Metrics

Miss Rate

- Fraction of memory references not found in cache (misses / references)
- Typical numbers: 3-10% for L1; can be quite small (e.g., $< 1\%$) for L2, depending on size, etc.

Hit Time

- Time to deliver a line in the cache to the processor (including time to determine whether the line is in the cache).
- Typical numbers: 1-3 clock cycles for L1; 5-12 clock cycles for L2.

Miss Penalty

- Additonal time required because of a miss.
- Typically 100-300 cycles for main memory.

Writing Cache Friendly Code

- Repeated references to variables are good (temporal locality).
- Stride-1 reference patters are good (spatial locality).

Examples:

Assume cold cache, 4-byte words, 4 word cache blocks.

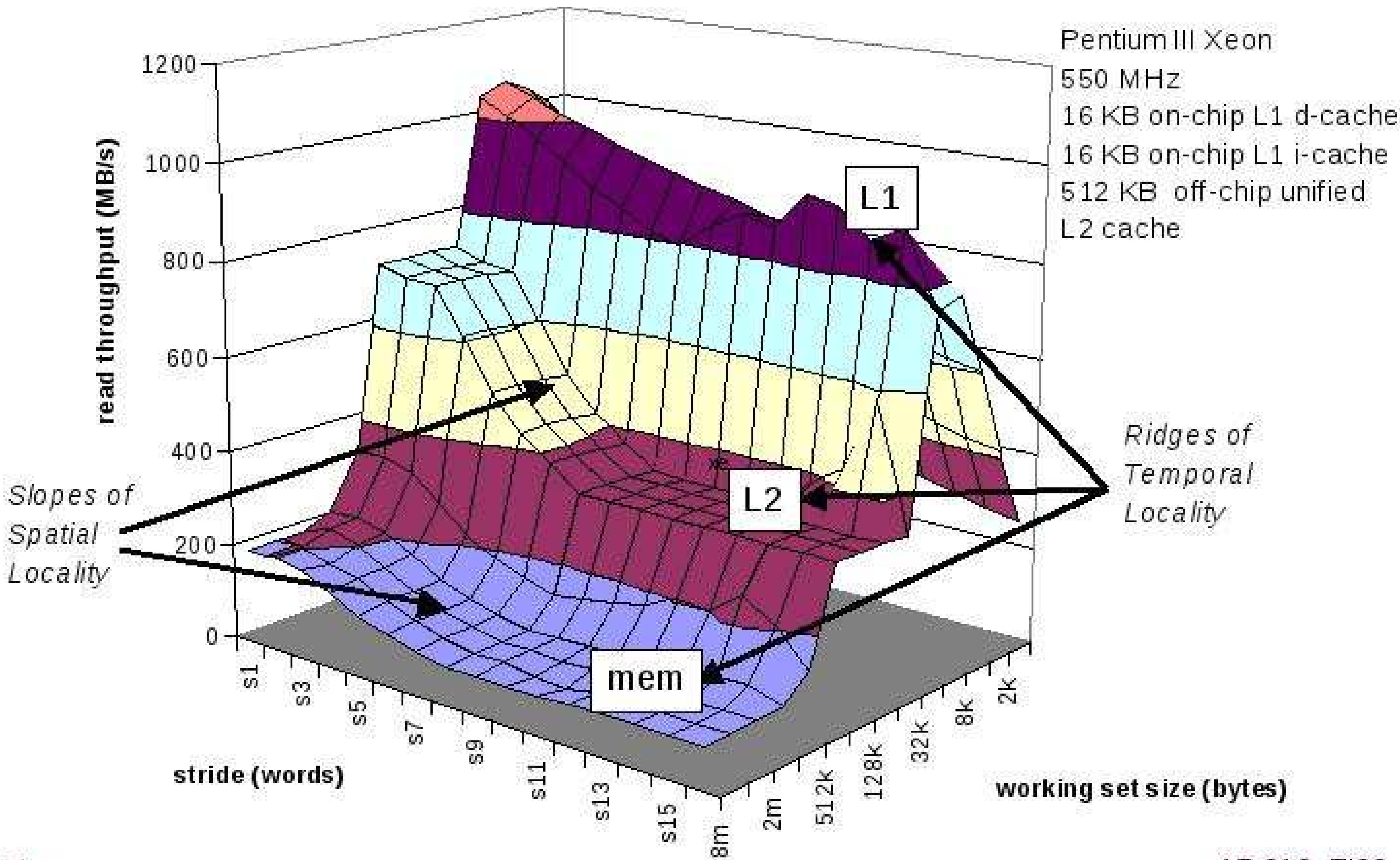
```
int sumarrayrows( int a[M][N]
    )
{
    int i, j, sum = 0;
    for( i = 0; i < M; i++ )
        for( j = 0; j < N; j++ )
            sum += a[i][j];
    return sum;
}
```

Miss rate $= 1/4 = 25\%$

```
int sumarraycols( int a[M][N]
    )
{
    int i, j, sum = 0;
    for( j = 0; j < N; j++ )
        for( i = 0; i < M; i++ )
            sum += a[i][j];
    return sum;
}
```

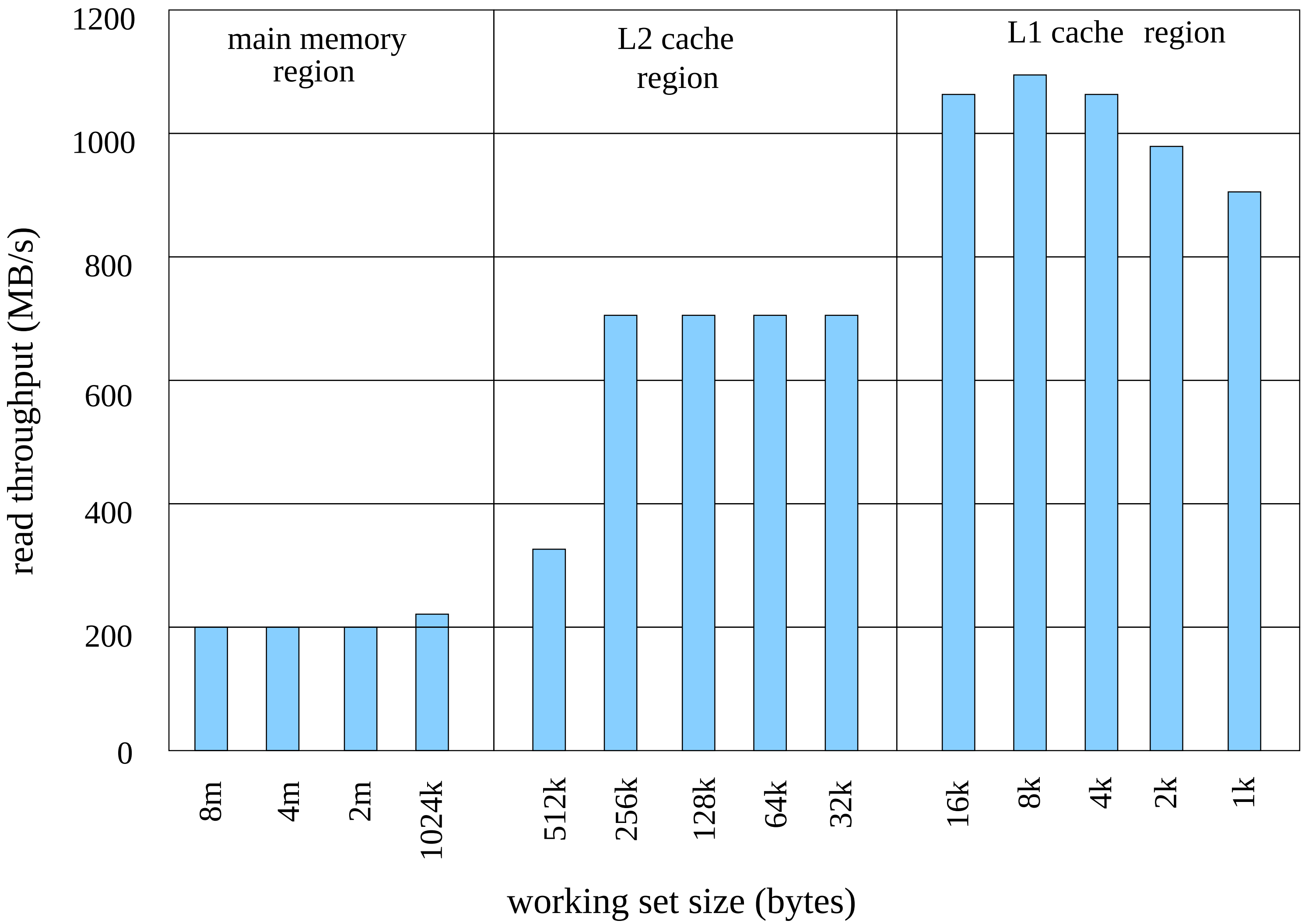
Miss rate $= 100\%$

The Memory Mountain



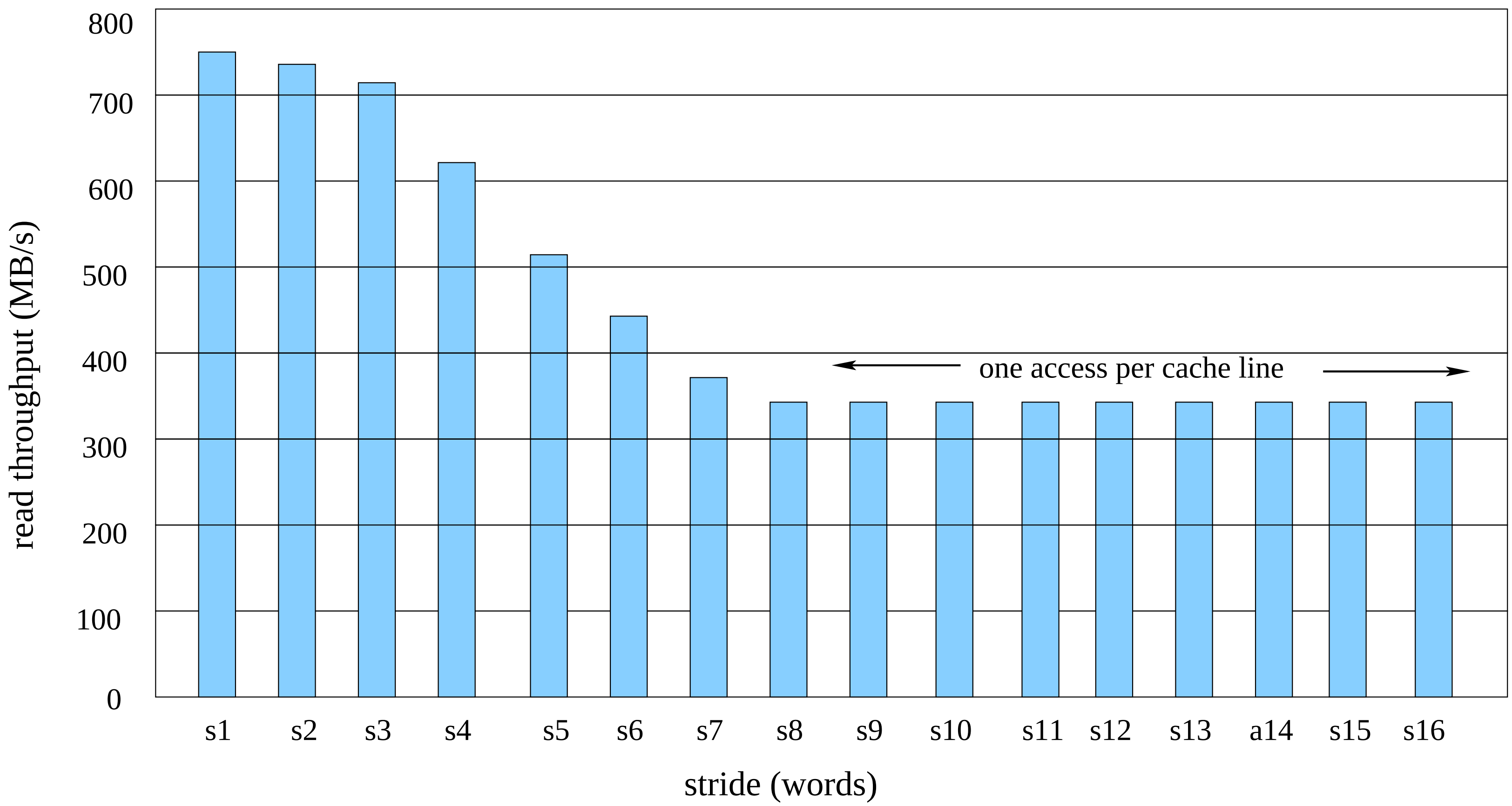
Slice through the memory mountain withn stride = 1.

This illustrates read throughput with different caches and memory.



A Slope of Spatial Locality

Slice through memory mountain with size = 256KB.
This shows cache block size.



Matrix Multiplication Example

Major Cache Effects to Consider.

- Total cache size: Exploit temporal locality and keep the working set small (e.g., by using blocking).
- Block size: Exploit spatial locality.

Description

- Multiply $N \times N$ matrices.
- $O(N^3)$ total operations.
- Accesses:
  - N reads per source element
  - N values summed per destination (but may be held in register).

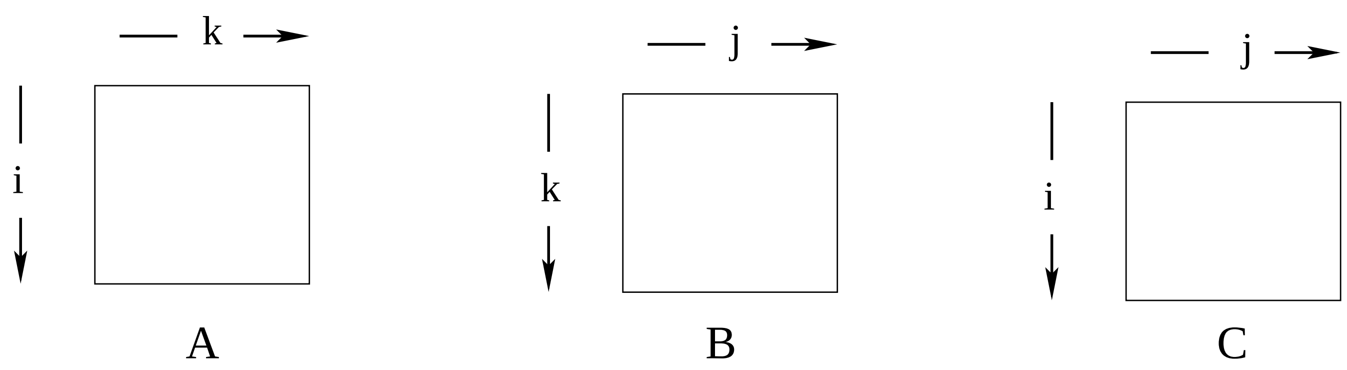
```c
/* ijk */
for (i=0; i<n; i++) {
  for (j=0; j<n; j++) {
    sum = 0.0;      // in reg
    for (k=0; k<n; k++)
      sum += a[i][k] * b[k][j];
    c[i][j] = sum;
  }
}
```

Miss Rate for Matrix Multiply

Assume:

- Line size = 32B (big enough for 4 64-bit words)
- Matrix dimension N is very large.
- We can approximiate $1/N$ as 0.0.
- Cache is not even big enough to hold multiple rows.

Analysis Method: Look at access pattern of the inner loop.



Layout of C Arrays in Memory (review)

C arrays are allocated in row-major order.

- Each row is allocated in contiguous memory locations.

Stepping through columns in one row:

```
for (i = 0; i < N; i++)
    sum += a[j][i];
```

- This accesses successive elements.
- If block size $B > 4$ bytes, exploits spatial locality.
- Compulsary miss rate = 4 bytes / B.

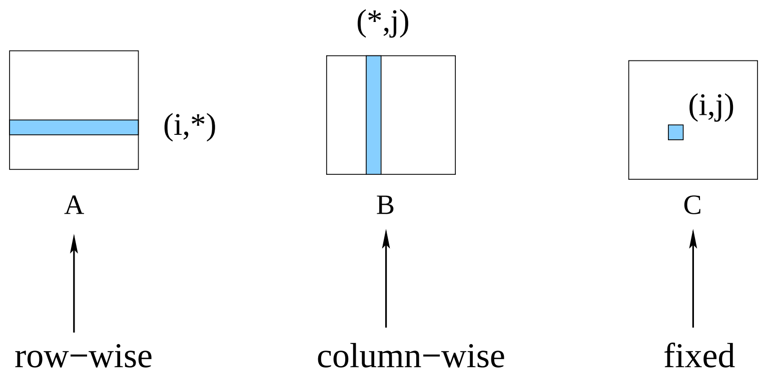
Stepping through rows in one column:

```
for (i = 0; i < N; i++)
    sum += a[i][i];
```

- Accesses distant elements.
- No spatial locality!
- Compulsary miss rate = 1 (i.e., 100%).

Matrix Multiplication (ijk)

```
/* ijk */
for (i=0; i<n; i++) {
  for (j=0; j<n; j++) {
    sum = 0.0;      // in reg
    for (k=0; k<n; k++)
      sum += a[i][k] * b[k][j];
    c[i][j] = sum;
  }
}
```

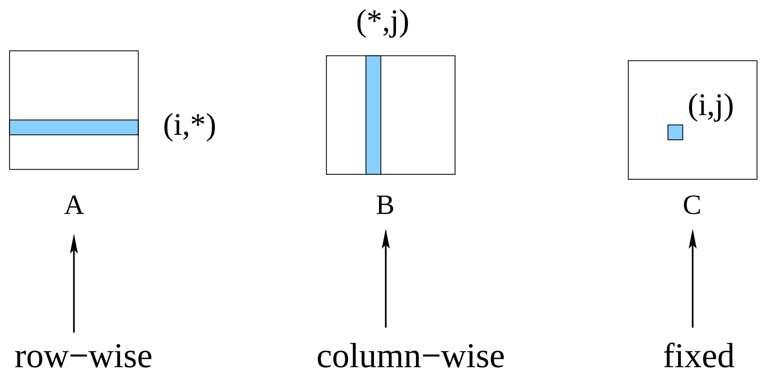


Misses per Inner Loop Iteration:

A	B	C
0.25	1.0	0.0

Matrix Multiplication (jik)

```
/* ijk */
for (j=0; j<n; j++) {
  for (i=0; i<n; i++) {
    sum = 0.0;        // in reg
    for (k=0; k<n; k++)
      sum += a[i][k] * b[k][j];
    c[i][j] = sum;
  }
}
```

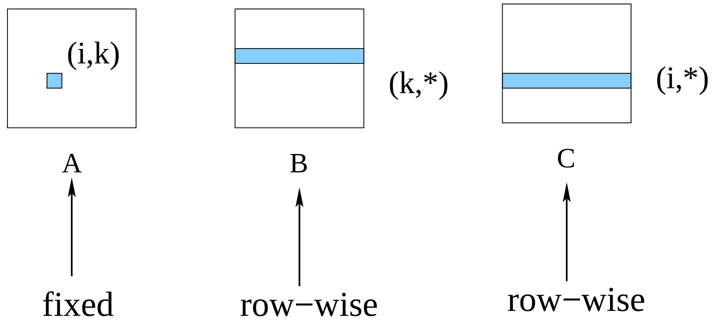


Misses per Inner Loop Iteration:

A	B	C
0.25	1.0	0.0

Matrix Multiplication (kij)

```
/* kij */
for (k=0; k<n; k++) {
  for (i=0; i<n; i++) {
    r = a[i][k];
    for (j=0; j<n; j++)
      c[i][j] += r * b[k][j];
  }
}
```

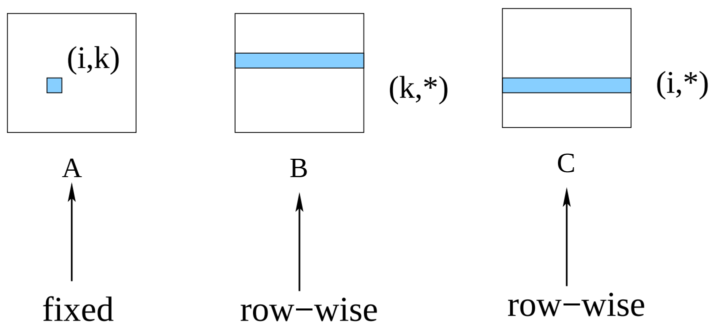


Misses per Inner Loop Iteration:

A	B	C
0.0	0.25	0.25

Matrix Multiplication (ikj)

```
/* ikj */
for (i=0; i<n; i++) {
  for (k=0; k<n; k++) {
    r = a[i][k];
    for (j=0; j<n; j++)
      c[i][j] += r * b[k][j];
  }
}
```

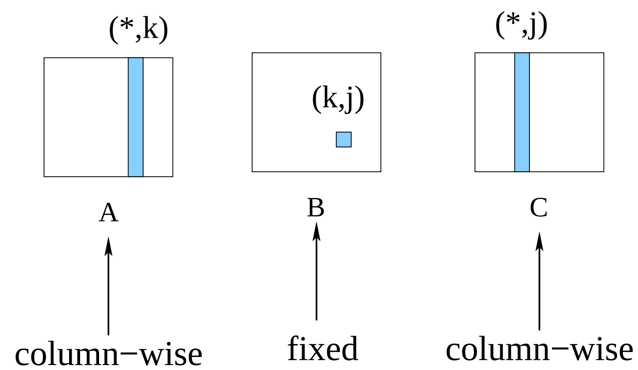


Misses per Inner Loop Iteration:

A	B	C
0.0	0.25	0.25

Matrix Multiplication (jki)

```
/* jki */
for (j=0; j<n; j++) {
  for (k=0; k<n; k++) {
    r = b[k][j];
    for (i=0; i<n; i++)
      c[i][j] += a[i][k] * r;
  }
}
```

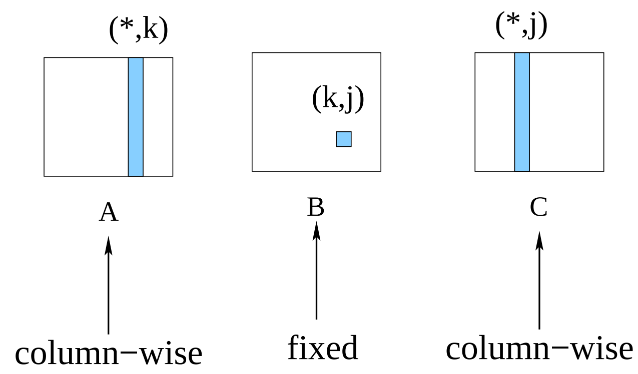


Misses per Inner Loop Iteration:

A	B	C
1.0	0.0	1.0

Matrix Multiplication (kji)

```
/* kji */
for (k=0; k<n; k++) {
  for (j=0; j<n; j++) {
    r = b[k][j];
    for (i=0; i<n; i++)
      c[i][j] += a[i][k] * r;
  }
}
```



Misses per Inner Loop Iteration:

A	B	C
1.0	0.0	1.0

Summary of Matrix Multiplication

ijk (& jik):

- 2 loads, 0 stores
- misses / iteration = 1.25

kij (& ikj):

- 2 loads, 1 store
- misses / iteration = 0.5

jki (& kji):

- 2 loads, 1 store
- misses / iteration = 2.0

Miss rates are important, but not perfect predictors of performance.. Code scheduling matters, also.

Improving Temporal Locality by Blocking

Example: Blocked matrix multiplication

- "Block" (in this context) does not mean "cache block."
- It means a sub-block within the structure (matrix).
- Example: $N = 8$; sub-block size $= 4$.

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \times \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}$$

Key idea: Sub-blocks (e.g., A_{xy}) can be treated just like scalars.

$$C_{11} = A_{11}B_{11} + A_{12}B_{21} \qquad C_{12} = A_{11}B_{12} + A_{12}B_{22}$$

$$C_{21} = A_{21}B_{11} + A_{22}B_{21} \qquad C_{22} = A_{21}B_{12} + A_{22}B_{22}$$

Blocked Matrix Multiply (bijk)

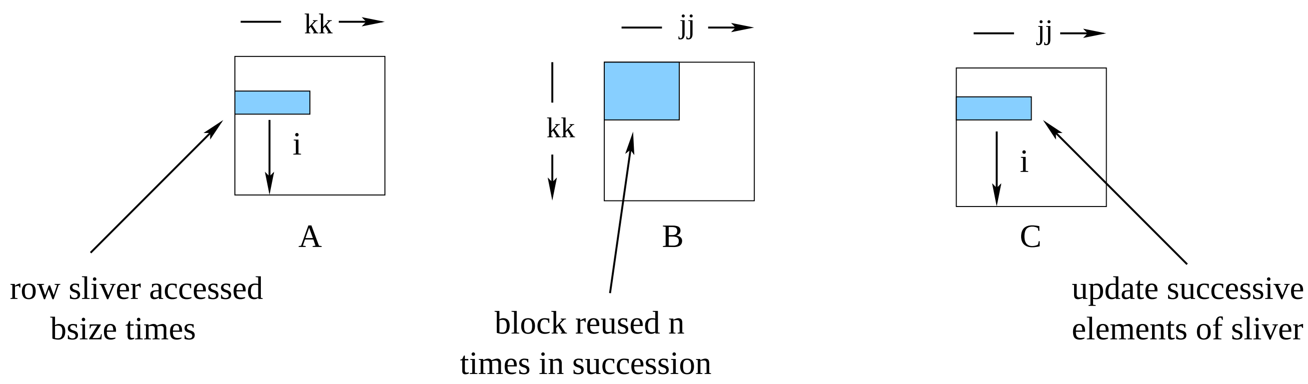
```
for (jj=0; jj<n; jj+=bsize) {
  for (i=0; i<n; i++)
    for (j=jj; j < min(jj+bsize,n); j++)
      c[i][j] = 0.0;
  for (kk=0; kk<n; kk+=bsize) {
    for (i=0; i<n; i++) {
      for (j=jj; j < min(jj+bsize,n); j++) {
        sum = 0.0;
        for (k=kk; k < min(kk+bsize,n); k++) {
          sum += a[i][k] * b[k][j];
        }
        c[i][j] += sum;
      }
    }
  }
}
```

Blocked Matrix Multiply Analysis

```
for (i=0; i<n; i++) {
  for (j=jj; j < min(jj+bsize,n); j++) {
    sum = 0.0;
    for (k=kk; k < min(kk+bsize,n); k++) {
      sum += a[i][k] * b[k][j];
    }
    c[i][j] += sum;
  }
}
```

Innermost loop pair multiplies a $1 \times bsize$ sliver of A by a $bsize \times bsize$ block of B and accumulates into a $1 \times bsize$ sliver of C.

Loop over i steps through n row slivers of A and C, using same B.



Blocked Matrix Multiply Performance

On a Pentium, blocking (bijk and bikj) improves peformance by a factor of two over the unblocked versions (ijk and jik).

The result is relatively insensitive to array size.

Concluding Observations

The programmer can optimize for cache performance.

- How data structures are organized.
- How data are accessed (e.g., nested loop structure).
- Blocking is a general technique.

All systems favor "cache friendly code."

- Getting absolute optimum performance is very platform specific.
- Involves cache sizes, line sizes, associativities, etc.
- Can get most advantage with generic code.
- Keep working set reasonably small (temporal locality).
- Use small strides (spatial locality).