Propositional Syllogisms

Whereas the logic of the categorical syllogism is based on the relationships between categories, the logic of the propositional syllogism is based on the relationships between propositions. We will study three kinds of propositional syllogisms here. Each kind is determined by the kind of connective which establishes the relationship between the propositions: Conjunction, Disjunction, and Implication (in hypothetical propositions).

Hypothetical Syllogisms

Two propositions may be joined into a compound proposition by using the words "if" and "then". The resulting compound proposition is known as a hypothetical or conditional proposition. In a hypothetical (in standard form) the component proposition which follows the "if" is called the antecedent, and the other, which follows the "then", is called the consequent. For example in

1.) "If might makes right, then love has no place in the world."

"might makes right" is the antecedent and "love has no place in the world" is the consequent. A hypothetical does not assert the truth of either its antecedent or its consequent. It asserts that a relationship exists between the two such that if the antecedent is true the consequent must be true.

Hypotheticals are not always stated in standard form. The order of the antecedent and the consequent may be switched as in this case:

2.) Love has no place in the world if might makes right.

Here the antecedent still follows the "if" and so it can easily be put into standard form by moving the "if" together with the antecedent to the beginning of the statement and adding "then" before the consequent. The word "unless" may be used to express a hypothetical as in this case:

3.) Love has no place in the world unless might does not make right.

In this case, one must notice that "unless" means the same as "if it is not the case that". So in standard form this proposition becomes "If it is not the case that might does not make right, then love has no place in the world." (The two negatives in the antecedent may be taken to cancel each other out and that would leave us with statement 1.) above.) One of the most confusing non-standard forms involves "only if" as in this case:

4.) Might makes right only if love has no place in the world.

These cases of "only if" are the only times when "if" does not introduce an antecedent, but rather a consequent. So when statement 4.) is put into standard form it becomes statement 1.); they mean the same. Do not confuse "only if" with "if only" as in this case:

5.) If only might made right, then love would have no place in the world.
Here the word "only" does no real logical work but merely serves to provide some emphasis. Thus statement 5.) says the same thing as statement 1.) (Hopefully, no one would actually assert statement 5.) as the connotations are different from 1.).)

Mixed Hypothetical Syllogisms

A mixed hypothetical syllogism consists of a hypothetical proposition as the first premise and a premise which either affirms or denies either the antecedent or the consequent. The conclusion will then either affirm or deny whichever was not used in the second premise. For example:

If love has a place in the world, then might does not make right.
Love has a place in the world.
Might does not make right.

Note that since hypothetical propositions do not assert either the antecedent or the consequent, the second premise really does add information to the argument: namely that the antecedent is true. In other words the second premise affirms the antecedent. This is the form of this argument: **affirming the antecedent**. This form is called **modus ponens**. Modus ponens is always valid; the premises could not be all true with a false conclusion. If the argument had affirmed the consequent, it would be a different argument with a different form:

If love has a place in the world, then might does not make right.
Might does not make right.
Love has a place in the world.

This form is called **affirming the consequent** and is invalid. (None of the information given rules out the possibility that love has no place in the world for some other reason, which is quite consistent with both premises being true.) If the second premises denies the consequent we have a third form of hypothetical syllogism:

If love has a place in the world, then might does not make right.
Might does make right.
Love has no place in the world.

This form, **denying the consequent**, is called **modus tollens** and is valid. Notice that the consequent is denied by making an affirmative proposition because the consequent makes a negative claim. To deny a claim is to claim the opposite: a negative claim if the original was affirmative, and affirmative if the original was negative. The final possible form is denying the antecedent:

If love has a place in the world, then might does not make right.
Love has no place in the world.
Might does make right.

This form is called **denying the antecedent** and is invalid. If we use letters to abbreviate whole propositions we can save a good deal of writing or
typing and at the same time make the differences between these four forms a little more obvious. We will use the following abbreviations:

\[
\begin{align*}
p &: \text{Love has a place in the world.} \\
q &: \text{Might does not make right.}
\end{align*}
\]

The four arguments then become as follows:

<table>
<thead>
<tr>
<th>Modus Ponens</th>
<th>Affirming the Consequent</th>
<th>Modus Tollens</th>
<th>Denying the Antecedent</th>
</tr>
</thead>
<tbody>
<tr>
<td>If p then q</td>
<td>If p then q</td>
<td>If p then q</td>
<td>If p then q</td>
</tr>
<tr>
<td>p</td>
<td>q</td>
<td>Not-q</td>
<td>Not-p</td>
</tr>
<tr>
<td>q</td>
<td></td>
<td>Not-p</td>
<td>Not-q</td>
</tr>
<tr>
<td>valid</td>
<td>invalid</td>
<td>valid</td>
<td>invalid</td>
</tr>
</tbody>
</table>

Note that if we chose different abbreviations the argument may look quite different and yet have the same relevant form. For example:

\[
\begin{align*}
p &: \text{Love has a place in the world.} \\
q &: \text{Might makes right.}
\end{align*}
\]

These abbreviations would turn the third argument into:

\[
\begin{align*}
\text{If } p \text{ then not-}q \\
q \\
\text{Not-p}
\end{align*}
\]

But this argument is still modus tollens because the second premise still denies the consequent.

**Pure Hypothetical Syllogism**

The following argument is valid:

\[
\begin{align*}
p &: \text{Might makes right.} \\
q &: \text{Love has no place in the world.} \\
r &: \text{Life is pointless.}
\end{align*}
\]

\[
\begin{align*}
\text{If } p \text{ then } q \\
\text{If } q \text{ then } r
\end{align*}
\]
If \( p \) then \( r \)

The order of the premises does not matter, so this argument is equally valid:

\[
\begin{align*}
\text{If } q \text{ then } r \\
\text{If } p \text{ then } q \\
\text{If } p \text{ then } r
\end{align*}
\]

The order of the propositions in the conclusion does matter. The antecedent in the conclusion must be an antecedent in its premise, and so also the consequent of the conclusion must be a consequent in its premise. The remaining proposition thus must occupy both positions, in one premise being the antecedent and in the other, the consequent. All other arrangements of form in a pure hypothetical syllogism are invalid.

**Disjunctive Syllogisms**

Two propositions may be joined into a compound proposition by using the word "or". The resulting proposition is called a disjunction and each component is called a disjunct. The order of the disjuncts does not matter (the meaning of a disjunction stays the same if one switches the disjuncts), so unique names for each position is unnecessary (unlike the hypothetical proposition with its antecedent and consequent). A disjunctive syllogism consists of a disjunction for the first premise and a premise which either affirms or denies a disjunct. For example:

Either UFO's exist or crop circles are a big hoax.
But UFO's do not exist.
Crop circles are a big hoax.

This argument has the form of **denying a disjunct** and is valid. Which disjunct happens to be denied cannot matter as the order of the disjuncts does not matter. An argument which **affirms a disjunct** is invalid, in spite of examples where it seems to work. The reason affirming a disjunct and then denying the other disjunct in the conclusion seems to work is that sometimes the word "or" is used in an **exclusive** sense. In the **exclusive** sense "or" excludes the possibility of both disjuncts being true. For example, either the colour of a pixel is black or it is some other colour. Since "or" is only sometimes used in this sense one cannot count on it, and so any conclusion which does count on it, has the possibility of being false even with true premises. In critical thinking we assume the **inclusive** sense of the word "or". In the **inclusive** sense "or" includes the possibility of both disjuncts being true.

<table>
<thead>
<tr>
<th>Denying a Disjunct</th>
<th>Denying a Disjunct</th>
<th>Affirming a Disjunct</th>
<th>Affirming a Disjunct</th>
</tr>
</thead>
<tbody>
<tr>
<td>Either ( p ) or ( q )</td>
<td>Either ( p ) or ( q )</td>
<td>Either ( p ) or ( q )</td>
<td>Either ( p ) or ( q )</td>
</tr>
<tr>
<td>( \neg p )</td>
<td>( \neg q )</td>
<td>( q )</td>
<td>( q )</td>
</tr>
<tr>
<td>( q )</td>
<td>( p )</td>
<td>( \neg q )</td>
<td>( \neg p )</td>
</tr>
<tr>
<td>valid</td>
<td>valid</td>
<td>invalid</td>
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</table>
If someone wishes to make a valid argument with the exclusive sense of "or" they should make the exclusion explicit by putting the argument in the form of the conjunctive syllogism.

**Conjunctive syllogisms**

When two propositions are joined by "and" into a compound proposition, the larger proposition is called a conjunction. Each component proposition is called a conjunct. The order of the conjuncts does not matter (the meaning of a conjunction stays the same if one switches the conjuncts), so unique names for each position is unnecessary (unlike the hypothetical proposition with its antecedent and consequent). A conjunctive syllogism consists of the denial of a conjunction for the first premise and a premise which either affirms or denies a conjunct. For example:

- It is not the case that both the colour of the pixel is black and the colour of the pixel is some other colour.
- The colour of the pixel is black.
- The colour of the pixel is not some other colour

The form of this syllogism is **affirming a conjunct**; it is valid. The form of **denying a conjunct** is invalid.

<table>
<thead>
<tr>
<th>Affirming a Conjunct</th>
<th>Affirming a Conjunct</th>
<th>Denying a Conjunct</th>
<th>Denying a Conjunct</th>
</tr>
</thead>
<tbody>
<tr>
<td>Not both p and q</td>
<td>Not both p and q</td>
<td>Not both p and q</td>
<td>Not both p and q</td>
</tr>
<tr>
<td>p</td>
<td>q</td>
<td>Not-p</td>
<td>Not-q</td>
</tr>
<tr>
<td>Not-q</td>
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<td>q</td>
<td>p</td>
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<td>valid</td>
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