Automatic Configuration of Mobile Conveyor Lines

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Abstract—A conveyor belt is an efficient mode of transportation and has been widely utilized to move large quantities of objects in assembly lines, airports, etc. We propose a new conveyor system called mobile conveyor lines that can autonomously configure itself to move objects to a given destination. This system is suitable for situations such as disaster areas in which it is difficult to set up a conveyor line manually. We analyze the reachability of a group of mobile conveyor belts and propose an algorithm to check the reachability of a given destination, as well as a method to generate a configuration to guide conveyor belts to connect themselves to reach the destination. Our experimental results show that our algorithms, together with a heuristic that biases the search towards the destination, can quickly generate configurations of conveyor belts for problems that require less than 20 conveyor belts.

I. INTRODUCTION

Many interesting tasks for mobile robots involve moving objects from one location to another. However, the design of existing mobile robots is not the best for moving a large number of objects as efficiently as possible. For example, the robots in the DARPA Robotics Challenge have limited ability to carry or transport a payload; they would have to make many round trips in order to move all objects out of a disaster zone in a rescue mission. While a robot with a high payload capacity can finish the task in a shorter amount of time, it makes more sense to deploy conveyor belts to help robots moving the objects. Conveyor technology, which has been widely used to move materials along manufacturing assembly lines or as a mode of transportation (e.g., escalators), could play a big part in high throughput robotic systems.

Today most conveyor belts are used in a stationary position in indoor environments (e.g., assembly lines). Few are designated as portable conveyor belts that can be deployed at outdoor locations by human workers. One example is Miniveyors’ portable conveyors as shown in Fig. 2. These lightweight conveyor systems allow quick and efficient transportation for a wide variety of materials, making them a good companion for robots in rescue missions. However, these conveyor belts still require humans to setup, making them difficult to be deployed in extreme environments such as disaster areas. We therefore propose to consider conveyor belts as robots and study conveyor belts with its own mobility. We call these robots mobile conveyor belts. In this paper, we present a mobile conveyor belt that is an ordinary conveyor belt attached to a mobile platform (see Fig. 1), and study how to make a number of these robots configure themselves autonomously without human intervention.

One challenge for our mobile conveyor system is how to connect several mobile conveyor belts together to form a conveyor line that transfers objects from one location to a given destination. Some destinations simply cannot be reached no matter how the conveyor belts are connected together. Moreover, as the number of conveyor belts increases, the number of possible ways to connect them grows exponentially, causing difficulty in computing the right configurations to form a conveyor line. To resolve these issues,

• we give a complete set of equations to describe the set of positions that can be reached by one mobile conveyor belt given its physical constraints;
• we present a probabilistic algorithm to check whether it is possible to use at most $N$ mobile conveyor belts to connect a position to a destination on a flat surface;
• we describe an approach to generate a correct configuration for all conveyor belts if the probabilistic algorithm shows that such configuration exists; and
• we propose a heuristic to increase the chance for the algorithms to find a solution in 3D environments.

This paper is organized as follows. After presenting the related work in Section II, we define our problem in Section III and analyze the reachability of a conveyor belt in Section IV and Section V. Then we give a configuration generation algorithm in Section VI and present the experimental results in Section VII, before we conclude in Section VIII.
II. RELATED WORK

Since robots typically cannot move items in large volumes, we consider incorporating some capabilities of robots into conveyor systems so that the robotic conveyor systems can be used in situations that no other robots can handle. There are few works concerning the configuration of conveyor systems as a whole. Instead, the majority of these works are about the improvement of mechanical design. Li and Li [1] used AMESIM, a hardware modeling software, to evaluate the performance of a conveyor belt and found that the addition of flywheels to motors can greatly improve the performance. Pitcher [2] discussed the loss of strength in three main types of joints between conveyor belts, and found that these joint design cannot perform at their full potential. Donis [3] studied the problem of how to make optimal choice of the locations of the belt weighter while taking the belt stiffness into account. Nuttall [4] studied the design of multiple driven belt conveyors, as well as distributed drive power and tension control, so as to strike a balance between locally applied drive power and the resulting resistances. Bindzar et al. [5] presented a 3D mathematical model of conveyor belt which is used to study its performance when subject to stress loading.

When conveyors are joined, they form a kinematic chain similar to a snake-like robot, which is a type of hyper-redundant manipulation whose configuration problem is mainly about finding collision-free configuration-space paths [14]. Our mobile conveyor line is similar to Job conveyor train, a continuous haulage system for paths [14]. Our mobile conveyor line is similar to Job conveyor train, a continuous haulage system for paths [14]. Our mobile conveyor line is similar to Job conveyor train, a continuous haulage system for paths [14]. Our mobile conveyor line is similar to Job conveyor train, a continuous haulage system for paths [14]. Our mobile conveyor line is similar to Job conveyor train, a continuous haulage system for paths [14]. Our mobile conveyor line is similar to Job conveyor train, a continuous haulage system for paths [14]. Our mobile conveyor line is similar to Job conveyor train, a continuous haulage system for paths [14]. Our mobile conveyor line is similar to Job conveyor train, a continuous haulage system for paths [14]. Our mobile conveyor line is similar to Job conveyor train, a continuous haulage system for paths [14]. Our mobile conveyor line is similar to Job conveyor train, a continuous haulage system for paths [14]. Our mobile conveyor line is similar to Job conveyor train, a continuous haulage system for paths [14]. Our mobile conveyor line is similar to Job conveyor train, a continuous haulage system for paths [14]. Our mobile conveyor line is similar to Job conveyor train, a continuous haulage system for paths [14]. Our mobile conveyor line is similar to Job conveyor train, a continuous haulage system for paths [14].

III. CONFIGURATIONS OF MOBILE CONVEYOR LINES

A conveyor belt is a machine that carries a continuous sequence of objects from one location to another. We attached a conveyor belt to a mobile platform as shown in Fig. 3. The mobile platform connects to the center of the conveyor belt via a strut with adjustable length. At the intersection of the strut and the conveyor belt is a servo motor that controls the slope of the conveyor belt. Let \( L \) be the length of the conveyor belt, \( h \) be the height of the center of the conveyor belt from the ground, and \( \theta \) be the pitch angle between the conveyor belt and the horizontal plane. Our mobile conveyor belt is height-adjustable and pitch-adjustable, with a minimum height \( H_{\text{min}} \), a maximum height \( H_{\text{max}} \), and a maximum pitch angle \( \Theta_{\text{max}} \) (i.e., \( H_{\text{min}} \leq h \leq H_{\text{max}} \) and \( -\Theta_{\text{max}} \leq \theta \leq \Theta_{\text{max}} \)). The maximum pitch angle are the same in both clockwise and counterclockwise directions, but the length of a conveyor belt is not adjustable.

A mobile conveyor line consists of \( n \) mobile conveyor belts linking together at their endpoints to form a chain of conveyor belts. To simplify analysis, we only consider mobile conveyor lines in which all mobile conveyor belts are identical; hence \( L, H_{\text{min}}, H_{\text{max}}, \) and \( \Theta_{\text{max}} \) are the same for all mobile conveyor belts. Let \( D_{\text{min}} \) and \( D_{\text{max}} \) be the minimum and maximum vertical distances between the endpoints of two consecutive belts, respectively. If there is a mechanism to physically connect the endpoints of two conveyor belts, we set \( D_{\text{min}} = D_{\text{max}} = 0 \); otherwise, we can stack one end point of a conveyor belt on top of the end point of another conveyor belt, as shown in Fig. 2. In the latter case, \( D_{\text{max}} \) is the maximum dropping distance the objects can sustain, and \( D_{\text{min}} > 0 \) because there is a gap between the endpoints of the two conveyor belts.

Let \( \tau_1, \tau_2, \ldots, \tau_N \) be \( N \) mobile conveyor belts that are put on a flat surface. Let \((x_i,y_i)\) be the coordinate of the center of \( \tau_i \), and \( \phi_i \) be the heading of the \( \tau_i \). Here the heading of the mobile platform is the same as the heading of the conveyor belt since our conveyor belt cannot rotate relative to the mobile belt. Let \( h_i \) and \( \theta_i \) be the height and the pitch angle of \( \tau_i \) as depicted in Fig. 3. A configuration of \( \tau_i \) is a 5-tuple \((x_i,y_i,\phi_i,h_i,\theta_i)\). Then the coordinate of the starting point of \( \tau_i \) is \((x_{\text{start}}, y_{\text{start}}) = (x_i - (L/2) \cos(\theta_i) \cos(\phi_i), y_i - (L/2) \cos(\theta_i) \sin(\phi_i), h_i - (L/2) \sin(\theta_i))\), and the coordinate of the end point of \( \tau_i \) is \((x_{\text{end}}, y_{\text{end}}) = (x_i + (L/2) \cos(\theta_i) \cos(\phi_i), y_i + (L/2) \cos(\theta_i) \sin(\phi_i), h_i + (L/2) \sin(\theta_i))\).

Objects enter the scene from an entry point, which has...
the coordinate \( p^{\text{entry}} = (x^{\text{entry}}, y^{\text{entry}}, z^{\text{entry}}) \), where \( z^{\text{entry}} \geq 0 \). Given \( N \) mobile conveyor belts, our goal is to connect \( n \) conveyor belts to form a conveyor line that moves objects entering from the entry point to an exit point \( p^{\text{exit}} = (x^{\text{exit}}, y^{\text{exit}}, z^{\text{exit}}) \), where \( z^{\text{exit}} \geq 0 \) and \( 0 \leq n \leq N \). In other words, our objective is to compute a set of configurations \( \{(x_i, y_i, \theta_i, h_i, \phi_i)\}_{i=1..n} \) for \( n \) mobile conveyor belts that satisfies the following constraints:

C1) \( x_i^{\text{end}} = x_i^{\text{start}} \) and \( y_i^{\text{end}} = y_i^{\text{start}} \) for \( 1 \leq i \leq n \);

C2) \( x_1^{\text{start}} = x_1^{\text{entry}} \) and \( y_1^{\text{start}} = y_1^{\text{exit}} \);

C3) \( D_{\text{min}} \leq \Delta z_i \) for \( 1 \leq i \leq n \);

C4) \( D_{\text{min}} \leq z^{\text{entry}} - z_i^{\text{start}} \) and \( D_{\text{min}} \leq z_i^{\text{end}} - z^{\text{exit}} \);

C5) \( h_i \leq H_{\text{max}} \) and \( \Theta_{\text{max}} \leq \theta_i \) for \( 1 \leq i \leq n \); and

C6) \( z_i^{\text{start}} \geq 0 \) and \( z_i^{\text{end}} \geq 0 \) for \( 1 \leq i \leq n \).

IV. REACHABILITY ANALYSIS

Due to its physical constraints, a mobile conveyor belt can connect a starting point to a subset of end points in the workspace only. We say these end points are feasible. Given a starting point \( p^{\text{start}} \), let \( F \) be the set of all feasible end points. The reachable set \( R \) of the conveyor belt is the set of points below \( F \) such that each point satisfies the dropping distance constraint, i.e., \( R = \{(x,y,z') : z - D_{\text{max}} \leq z' \leq z - D_{\text{min}}, (x,y,z) \in F \} \). If a point \( p \) is not in \( R \), it means that it is impossible to configure a conveyor belt to move an object from \( p^{\text{start}} \) to some end point \( p^{\text{end}} \) and then drop the object to \( p \). As we will see in Section VI, computing the reachable set given a starting point can help to determine if a certain configuration of a conveyor line is feasible. In this section, we will give a complete set of equations to describe the reachable set of one conveyor belt only.

To simplify our discussion, we begin by considering the 2D reachable set on the \( x-z \) plane by setting \( y = 0 \) and \( \phi = 0 \). We can easily extend this 2D reachable set to 3D by rotating the 2D reachable set along the vertical line passing through the starting point of the conveyor belt—then we get a toroidal region which is the reachable set of the conveyor belt in 3D. Without loss of generality, suppose the starting point of the conveyor belt is fixed at \((0,0,z^{\text{start}})\). The following equation defines the reachable set in terms of \( \theta \):

\[
R(\theta) = \{(x,0,z) : x = L\cos(\theta), z^{\text{start}} + L\sin(\theta) - D_{\text{max}} \leq z \leq z^{\text{start}} + L\sin(\theta) - D_{\text{min}}, z \geq 0 \}.
\]

This reachable set is a vertical line segment whose position and length depend on the value of \( \theta \). Let \( \Theta = [\Theta_\text{min}, \Theta_\text{max}] \) be the valid range of \( \theta \) such that the conveyor belt satisfies all physical constraints. The reachable set is then

\[
R = \bigcup_{\Theta_\text{min} \leq \theta \leq \Theta_\text{max}} R(\theta).
\]

The valid range of \( \theta \) depends on the values of \( H_{\text{min}}, H_{\text{max}}, \Theta_{\text{max}}, L \), and \( z^{\text{start}} \). Fig. 4 lists the six cases in which the set of valid range of \( \theta \) differs. In this figure, \( \Delta_{\text{max}} = (L/2)\sin(\Theta_{\text{max}}) \) is the maximum vertical distance between the starting point and the center of the belt. In essence, the six cases show how \( H_{\text{min}}, H_{\text{max}}, \Theta_{\text{max}}, \) and \( L \) relate to each other. The range of \( \theta \) in each case depends on the value of \( z^{\text{start}} \) as shown in Table I. Some constants in Table I are defined in Table II.

To see why the cases in Table I are exhaustive, we need to understand how we identify the six cases in Fig. 4. In Case 1 and Case 2, \( H_{\text{min}} \) is large enough so that the conveyor belt cannot touch the ground. The red lines represent the conveyor belt when \( h = H_{\text{max}} \) at the minimum and maximum pitch angles; Point A and Point B are the starting points of the belt at the minimum and maximum pitch angles, respectively. Likewise, the blue lines represent the conveyor belt when \( h = H_{\text{min}} \) at the minimum and maximum pitch angles, whereas Point C and Point D are the corresponding starting points. In Case 1, \( z^{\text{start}} \) is between Point A and Point B (i.e., \( z_B \leq z^{\text{start}} \leq z_A \) where \( z_A \) and \( z_B \) are the \( z \)-coordinates of A and B, respectively), the maximum \( \theta \) is limited by \( H_{\text{max}} \) while the minimum \( \theta \) has no limitation except \( \Theta_{\text{max}} \).

Hence, the valid range of \( \theta \) is \([-\Theta_{\text{max}}, \Theta_{\text{upper}}]\), where \( \Theta_{\text{upper}} = \arcsin((H_{\text{max}} - z^{\text{start}})/(L/2)) \) is the angle at which the conveyor belt is at the maximum height. Similarly, when \( z^{\text{start}} \) is between Point C and Point D, the minimum \( \theta \) is limited by \( H_{\text{min}} \), while the maximum \( \theta \) has no limitation except \( \Theta_{\text{max}} \). Hence, the valid range of \( \theta \) is \([\Theta_{\text{lower}}, \Theta_{\text{max}}]\), where \( \Theta_{\text{lower}} = \arcsin((H_{\text{min}} - z^{\text{start}})/(L/2)) \) is the angle at
TABLE I: The range of $\theta$ in every case in Fig. 4

<table>
<thead>
<tr>
<th>Case</th>
<th>Range of $z_{\text{start}}$</th>
<th>Range of $\theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1a</td>
<td>$z_{\text{start}} \leq z_{\text{start}} \leq z_{\text{end}}$</td>
<td>$-\Theta_{\text{max}} \leq \theta \leq \Theta_{\text{upper}}$</td>
</tr>
<tr>
<td>1b</td>
<td>$z_{\text{start}} \leq z_{\text{start}} \leq z_f$</td>
<td>$-\Theta_{\text{max}} \leq \theta \leq \Theta_{\text{max}}$</td>
</tr>
<tr>
<td>1c</td>
<td>$z_f \leq z_{\text{start}} \leq z_{\text{end}}$</td>
<td>$\Theta_{\text{lower}} \leq \theta \leq \Theta_{\text{max}}$</td>
</tr>
<tr>
<td>2a</td>
<td>$z_{\text{start}} \leq z_{\text{start}} \leq z_{\text{end}}$</td>
<td>$-\Theta_{\text{max}} \leq \theta \leq \Theta_{\text{upper}}$</td>
</tr>
<tr>
<td>2b</td>
<td>$z_{\text{start}} \leq z_{\text{start}} \leq z_f$</td>
<td>$-\Theta_{\text{max}} \leq \theta \leq \Theta_{\text{max}}$</td>
</tr>
<tr>
<td>2c</td>
<td>$z_f \leq z_{\text{start}} \leq z_{\text{end}}$</td>
<td>$\Theta_{\text{lower}} \leq \theta \leq \Theta_{\text{max}}$</td>
</tr>
<tr>
<td>3a</td>
<td>$z_{\text{start}} \leq z_{\text{start}} \leq z_{\text{end}}$</td>
<td>$-\Theta_{\text{max}} \leq \theta \leq \Theta_{\text{upper}}$</td>
</tr>
<tr>
<td>3b</td>
<td>$z_{\text{start}} \leq z_{\text{start}} \leq z_f$</td>
<td>$-\Theta_{\text{max}} \leq \theta \leq \Theta_{\text{max}}$</td>
</tr>
<tr>
<td>3c</td>
<td>$z_f \leq z_{\text{start}} \leq z_{\text{end}}$</td>
<td>$\Theta_{\text{lower}} \leq \theta \leq \Theta_{\text{max}}$</td>
</tr>
<tr>
<td>3d</td>
<td>$z_f \leq z_{\text{start}} \leq z_f$</td>
<td>$\Theta_{\text{lower}} \leq \theta \leq \Theta_{\text{upper}}$</td>
</tr>
<tr>
<td>4a</td>
<td>$z_{\text{start}} \leq z_{\text{start}} \leq z_{\text{end}}$</td>
<td>$-\Theta_{\text{max}} \leq \theta \leq \Theta_{\text{max}}$</td>
</tr>
<tr>
<td>4b</td>
<td>$z_{\text{start}} \leq z_{\text{start}} \leq z_f$</td>
<td>$\Theta_{\text{lower}} \leq \theta \leq \Theta_{\text{upper}}$</td>
</tr>
<tr>
<td>4c</td>
<td>$z_f \leq z_{\text{start}} \leq z_{\text{end}}$</td>
<td>$\Theta_{\text{lower}} \leq \theta \leq \Theta_{\text{max}}$</td>
</tr>
<tr>
<td>4d</td>
<td>$z_f \leq z_{\text{start}} \leq z_f$</td>
<td>$\Theta_{\text{lower}} \leq \theta \leq \Theta_{\text{upper}}$</td>
</tr>
<tr>
<td>5a</td>
<td>$z_{\text{start}} \leq z_{\text{start}} \leq z_{\text{end}}$</td>
<td>$-\Theta_{\text{max}} \leq \theta \leq \Theta_{\text{max}}$</td>
</tr>
<tr>
<td>5b</td>
<td>$z_{\text{start}} \leq z_{\text{start}} \leq z_f$</td>
<td>$\Theta_{\text{lower}} \leq \theta \leq \Theta_{\text{upper}}$</td>
</tr>
<tr>
<td>5c</td>
<td>$z_f \leq z_{\text{start}} \leq z_{\text{end}}$</td>
<td>$\Theta_{\text{lower}} \leq \theta \leq \Theta_{\text{max}}$</td>
</tr>
<tr>
<td>5d</td>
<td>$z_f \leq z_{\text{start}} \leq z_f$</td>
<td>$\Theta_{\text{lower}} \leq \theta \leq \Theta_{\text{upper}}$</td>
</tr>
<tr>
<td>6a</td>
<td>$z_f \leq z_{\text{start}} \leq z_f$</td>
<td>$\Theta_{\text{floor}} \leq \theta \leq \Theta_{\text{upper}}$</td>
</tr>
<tr>
<td>6b</td>
<td>$z_f \leq z_{\text{start}} \leq z_f$</td>
<td>$\Theta_{\text{lower}} \leq \theta \leq \Theta_{\text{upper}}$</td>
</tr>
</tbody>
</table>

In Table I, $\Delta_{\text{max}}$ is the maximum change in $z$ from the starting to the ending point, $\Theta_{\text{max}}$ is the maximum pitch angle, $\Theta_{\text{upper}}$ is the upper bound of $\theta$, $\Theta_{\text{lower}}$ is the lower bound of $\theta$, and $\Theta_{\text{floor}}$ is the minimum pitch angle.

TABLE II: Definitions of the constants in Fig. 4 and Table I.

- $\Delta_{\text{max}} = (L/2) \sin(\Theta_{\text{max}})$
- $\Theta_{\text{lower}} = \arcsin((H_{\text{min}} - z_{\text{start}})/(L/2))$
- $\Theta_{\text{upper}} = \arcsin((H_{\text{max}} - z_{\text{start}})/(L/2))$
- $\Theta_{\text{floor}} = \arcsin(z_{\text{start}}/L)$

Fig. 5: Some reachable sets in Case 1 and Case 2. We have $H_{\text{min}} = 5m$ in Case 1 and $H_{\text{min}} = 16m$ in Case 2. Other parameters are $L = 10m$, $H_{\text{max}} = 20m$, $D_{\text{min}} = 2m$, $D_{\text{max}} = 5m$, and $\Theta_{\text{max}} = 30^\circ$.

There is one more special point on the z-axis in this case: Point G is the starting point when $h = H_{\text{max}}$ and the ending point touches the ground (the magenta line in Fig. 4), $z_{\text{start}}$ cannot be larger than $z_G$: otherwise the ending point will go below the ground. The upper bound of $\theta$ is always bound by $H_{\text{max}}$, hence $\theta \leq \Theta_{\text{upper}}$. The lower bound of $\theta$ depends on whether $z_{\text{start}} \geq z_F$.

In Fig. 4 we can see that the line $z = H_{\text{max}}$ gradually approaches the line $z = H_{\text{min}}$ from Case 3 to Case 6. Case 5 is the last case we need to consider because $H_{\text{max}}$ cannot be less than $H_{\text{min}}$. Given $H_{\text{min}}$, $H_{\text{max}}$, $\Theta_{\text{max}}$, $L$, and $z_{\text{start}}$, we can find the upper bound and the lower bound of $\theta$ from Table I and Fig. 4. Using $\theta$ bounds in Eq. 2 yields the reachable set. Fig. 5 shows some examples of the reachable sets (the orange regions) in Case 1 and Case 2. Fig. 5(c) shows the constraint $z \geq 0$ limiting the reachable set.

V. GENERATING CONFIGURATIONS FOR A REACHABLE POINT

Given a point $p = (x, y, z) \in R$ which is reachable from $p_{\text{start}} = (x_{\text{start}}, y_{\text{start}}, z_{\text{start}})$, we want to compute the configuration $(x', y', \phi, h, \theta)$ for a conveyor belt to reach $p$. In our 2D environments, $y' = y_{\text{start}} = 0$ and $\phi = 0$. Since $p_{\text{start}}$ is fixed, we have $x' = x_{\text{start}} + (L/2) \cos(\theta)$ and $h = z_{\text{start}} + (L/2) \sin(\theta)$, both of them depend on $\theta$. To compute $\theta$, consider 1) the vertical line segment $[z + D_{\text{min}}, z + D_{\text{max}}]$ at $x$, and 2) the arc of the circle centered at $(x_{\text{start}}, 0, z_{\text{start}})$ with a radius $L$ and a range of angles of the maximum and minimum of $\theta$ according to Table I. There are either one
or two intersections between the line segment and the arc. These intersections are the end points of the conveyor belt that satisfy the dropping distance constraint to reach \((x, 0, z)\). Each intersection can yield a value of \(\theta\) by \(\theta = \arcsin((z'' - z^{\text{start}})/L)\) where \(z''\) is the height of the intersection. Thus there can be two possible configurations to reach \((x, 0, z)\).

We can easily extend this calculation to any reachable point \((x, y, z)\) in the 3D toroidal reachable set in 3D environments by transforming \(p^{\text{start}}\) and \((x, y, z)\) to the x-z plane by a rotation matrix. Then the configuration is \((x^{\text{start}} + (L/2)\cos(\theta), y^{\text{start}} \frac{\sin(\theta)}{\sqrt{1 - \cos^2(\theta)}}, z^{\text{start}} + (L/2)\sin(\theta), \theta)\) for one or two possible values of \(\theta\).

VI. THE AUTOMATIC CONFIGURATION ALGORITHM AND THE OVERLAPPING EFFECT

Based on the reachability analysis in Section IV, we devised Algorithm 1 to generate a configuration for a set of conveyor belts to connect an entry point \(p^{\text{entry}} = (x^{\text{entry}}, y^{\text{entry}}, z^{\text{entry}})\) to an exit point \(p^{\text{exit}} = (x^{\text{exit}}, y^{\text{exit}}, z^{\text{exit}})\). The algorithm starts from \(p^{\text{entry}}\) and considers adding conveyor belts one at a time to the conveyor line and randomly computing a set of \(M\) reachable sets that the newly added conveyor belts may reach, until it finds a reachable set that contains \(p^{\text{exit}}\) (Line 8). We denote the set of reachable sets after adding the i'th conveyor belt as \(Q^i\). Initially, we set \(Q^0\) to contain only one reachable set, which is the vertical line segment below \(p^{\text{entry}}\) (Line 2–3). For \(i \geq 1\), \(Q^i\) is computed by 1) computing the union \(U_i\) of the all reachable sets in \(Q^{i-1}\) (Line 5), 2) randomly choosing \(M\) points in \(U_i\) (Line 6), and 3) finding the reachable sets of the chosen points according to Section IV and adding them to \(Q^i\) (Line 7). Suppose a reachable set that contains \(p^{\text{exit}}\) is found after adding \(n\) conveyor belts. After that the algorithm searches backward to find the sequence of reachable sets that leads to \(p^{\text{exit}}\), as shown in the red regions in Fig. 6. The points \(p_0^*, p_1^*, \ldots, p_{n-1}^*\) from which the reachable sets are generated (i.e., the yellow dots in Fig. 6) are the starting points of the \(n\) conveyor belts (Line 12–13). Since the algorithm keeps all \(p_k\) and \(R_k^i\) generated in Line 7, no computation is needed to find \(p_{n-1}^*\) in Line 13. Then we can compute a configuration for the entire conveyor line using \(p_0^*, p_1^*, \ldots, p_{n-1}^*\) according to Section V (Line 14–16). We can easily verify that this configuration satisfies the constraints C1–C6 in Section III.

Algorithm 1 works in both 2D and 3D environments. In 2D environments, all conveyor belts are aligned on a 2D plane. As shown in Fig. 6, there are many overlaps among reachable sets. This is true in practice because the maximum pitch angles and maximum dropping distances are usually quite small, constraining the possible starting points of the next conveyor belt to a small region. We call this observation the overlapping effect. Our algorithm exploits this effect by sampling in the union of the set of reachable sets uniformly in Line 5–6. As we will demonstrate in our experiment in Section VII, searching with \(M\) sampling points simultaneously is better than making \(M\) independent searches, each with one sample point. This is because the overlapping region will be less likely to be sampled again due to the overlapping effect.

However, the overlapping effect is less prominent in 3D environments. Due to the freedom in the extra dimension, the algorithm performs poorly if we do not guide the search towards the exit point. Our solution to this problem is to give a higher chance to the sample points that are closer to the exit point. More specifically, in Line 6, we randomly choose \(M \times K\) points in \(U_i\) for some constant \(K\). Then we assign a weight \(W_p\) to each chosen point \(p\), where \(W_p\) is a large constant \(W\) minus the distance between \(p\) and the exit point. After that we randomly select \(M\) points out of these chosen points according to their weights, such that points closer to the exit point will have a higher probability to be sampled. As shown in the next section, this heuristic can help finding a solution in 3D environments quickly.

VII. EXPERIMENTAL EVALUATION

We conducted two experiments to evaluate our automatic configuration algorithm in 2D and 3D environments. In the 2D experiment, for each \(1 \leq N \leq 20\), we randomly generated 100 solvable problems in a 2D environment by choosing the parameters with a uniform probability distribution in the following ranges: \(H_{\text{max}} \in [400, 600]\), \(H_{\text{min}} \in [200, 400]\), \(L \in [200, 400]\), \(D_{\text{max}} \in [30, 50]\), \(D_{\text{min}} \in [5, 10]\), \(\Theta_{\text{max}} = 30^\circ\). Here, all units except \(\Theta_{\text{max}}\) are in centimeter. For each set of parameters, we chose an entry point \(p^{\text{entry}}\) by setting \(x^{\text{entry}} = y^{\text{entry}} = 0\) and choosing \(z^{\text{entry}} \in [H_{\text{min}} - (L/2)\sin(\Theta_{\text{max}}), H_{\text{max}} + (L/2)\sin(\Theta_{\text{max}})]\). Starting
As before, we generated 100 problems for each of the problem generation increases. The success rates decrease gradually as the number of conveyor belts in an environment increases. As we predicted in Section VI, the algorithm has a higher success rate when \( M \) is large. However, in all cases the successful rates decrease gradually as the number of conveyor belts in a 3D environment, and thus the differences in success rates for different values of \( M \) is not as large as in the 2D experiment.

VIII. CONCLUSIONS AND FUTURE WORK

We proposed to incorporate mobility into conveyor belts and studied how to connect several mobile conveyor belts to form a conveyor line to reach a given destination. Conveyor belts are good at moving a large quantities of objects, and hence can play a larger role in any robotic system in rescue missions or logistic domains. Our key results include a complete set of equations to describe the reachable set of a mobile conveyor belt on a flat surface, which leads to an efficient probabilistic approach for automatic configuration. Our experimental results demonstrated the overlapping effect, which states that the reachable sets are often overlapped. In the future, we would like to study the automatic configuration of mobile conveyor lines subject to limited mobility in rough terrains such as nuclear disaster areas.

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