
Assignment 3

1. Show that

$$\frac{\partial \ln|A|}{\partial A} = (A^{-1})^T$$

2. Show that the eigenvalues of a positive definite matrix are positive.
3. Show that

$$(A + BD^{-1}C)^{-1} = A^{-1} - A^{-1}B(d + CA^{-1}B)^{-1}CA^{-1}$$

Hint: Start by multiplying both sides by $(A + BD^{-1}C)$.

4. Use the method of Lagrange multipliers to find the dimensions of the can of radius r and height h that has maximum volume given a fixed surface area A .
5. Consider the one-dimensional problem of accelerating a cart on a track. The goal is to pick a control law $u(t)$ that will get the car as far as possible down the track at time T , but at the same time avoid costly accelerations.

The dynamic equation is:

$$\ddot{x} = u(t)$$

with initial conditions

$$x(0) = 0$$

$$\dot{x}(0) = 0$$

and the cost functional is

$$J = x(T) - \frac{1}{2} \int_0^T u^2(t) dt$$

Use Euler-Lagrange to solve for the best $u(t)$ analytically.