A Simple Game

Alice can pick “cooperate” (C) or “defect” (D)

... so can Bob
Logic dictates C,C, which is a Nash equilibrium
Prisoner’s dilemma

Reward = years taken off prison sentence
IPD

Previous analyses assume one round of play

Things change if there are ongoing rounds

Scenario A: TFT plays TFT

Scenario B: ALL-D plays TFT

A

The cooperator will get

\[ 3 + 3\gamma + 3\gamma^2 + 3\gamma^3 + \cdots = \frac{3}{1 - \gamma} \]

B

And the defector will get

\[ 5 + \gamma + \gamma^2 + \gamma^3 + \cdots = 5 + \frac{\gamma}{1 - \gamma} \]
Evolution of Cooperation - Axelrod

A genetic algorithm plays IPD… and wins
# Work and Shirk

<table>
<thead>
<tr>
<th></th>
<th>Boss</th>
<th>Don’t Inspect</th>
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</thead>
<tbody>
<tr>
<td><strong>Work</strong></td>
<td>P-W-I</td>
<td>P-W</td>
</tr>
<tr>
<td><strong>Worker</strong></td>
<td>-I</td>
<td>-W</td>
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<tr>
<td><strong>Shirk</strong></td>
<td>0</td>
<td>W</td>
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**Lesson:**
Both Boss and Worker should use probabilistic policies.
Hauert: Change the IPD game to allow a third option

N players at a time

*Loners* refuse and get a standard payoff $\sigma = 1$

S remaining players consist of $n_c$ cooperators and $S-n_c$ defectors

If $S=1$, player has to be a loner

- Defectors payoff is: $P_d = r \frac{n_c}{S}$
- Cooperators payoff is: $P_c = P_d - 1$
$r < 2$

- cooperate
- opt out
- defect

$\Rightarrow$

$r > 2$

A

B
Bottom panels: 80% of sites adopt the strategy of a neighbor with probability proportional to the payoff difference.

Top two panels: adopt the strategy of the best neighbor in 3x3.
Another way [Zhu]:
Decide whether to raise the cooperation level in a fractional strategy

\[
p = \frac{1}{1 + \frac{3\alpha}{1-\delta}}
\]
Opponent is HCE

Opponent is ALL-C