Workshop III - SDP Workshop

September 19, 2018

The workshop will be held on September 24th. There are three problems and presentations should be at most 20 minutes.

1 Max-2-SAT

Recall that in an instance of Max-2-SAT we have a set of n $\{-1, 1\}$ variables, and m constraints of the form

$$C_i \equiv b_{i1}x_{i1} + b_{i2}x_{i2} \neq -2$$

where the x_{ij} are variables and the $b_{ij} \in \{-1, 1\}$ are signs fixed in the instance. The objective is satisfy as many constraints as possible. The trivial algorithm of choosing a random assignment achieves a $\frac{3}{4}$ approximation. Adapt the approximation algorithm we saw in class for Max-Cut to achieve an α approximation algorithm for some constant $\alpha > 0.75$.

2 Unique Games Approximation

Recall that an instance of Unique Label cover consists of a bipartite graph (L, R, E), a finite alphabet of labels Σ, and a set of permutations $\{f_e : \Sigma \to \Sigma \mid e \in E\}$. A labelling is a function $l : L \cup R \to \Sigma$. An edge $e = (u, v)$ is satisfied if $f_e(l(u)) = l(v)$. Our objective is to find an l so as to maximize the fraction of edges satisfied. Give an SDP+rounding based algorithm which given an instance which is $1 - \epsilon$ satisfiable (that is some assignment satisfies $1 - \epsilon$ fraction of edges) can expect to satisfy $1 - poly(k, 1/\epsilon)$ fraction of edges, where k is the number of labels $|\Sigma|$.

3 Lovász Sandwich Theorem

Let $G = (V, E)$ be an undirected graph. Let the **independence number**, $\alpha(G)$, be the size of the largest independent set in G. Let the **clique cover number**, $\overline{\chi}(G)$, be the size of the smallest partition of vertices of G into disjoint cliques.

We define $\vartheta(G)$ as the optimum of the following semidefinite program:

$$\begin{aligned} \text{minimize} \quad & k \\ \text{subject to} \quad & \langle v_i, v_j \rangle = \frac{-1}{k-1},\ (i, j) \notin E, \\ & \langle v_i, v_i \rangle = 1,\ \forall i \in V. \end{aligned}$$

First, explain why the above semidefinite program is a relaxation of the clique cover problem. Conclude that $\vartheta(G) \leq \overline{\chi}(G)$.

Next, show that $\alpha(G) \leq \vartheta(G)$, conclude that $\alpha(G) \leq \vartheta(G) \leq \overline{\chi}(G)$. (**Hint:** consider an optimal collection of v_i for the above SDP.)