1. How many ways can the positive integer $n$ be expressed as an ordered sum of (any number of) positive integers? For example, here are four of the several ways to express 4: 4, 3+1, 1+3, and 2+1+1.

2. How many non-decreasing functions are there from $[n]$ to $[m]$?

3. How many monic polynomials (polynomials with leading coefficient 1) of degree $n$ over the field $\mathbb{F}_p$, $p$ prime, never take the value 0?

4. A tournament is a directed graph $T = (V, E)$ with no self loops such that for all $v \neq w \in V$, exactly one of $(v, w)$ and $(w, v)$ is in $E$. $T$ is transitive if there is a permutation $\pi$ of $V = [n]$ such that $(v, w) \in E \iff \pi(v) < \pi(w)$. Show that there exists a tournament on $n$ vertices which has no transitive subtournament on $\lceil 2 \log_2 n \rceil + 1$ vertices.

5. An $(N, M, D, K, \epsilon)$-disperser is a bipartite graph $([N], [M], E)$ such that every node in $[N]$ has degree at most $D$, and for every subset $S \subseteq [N]$ of $K$ nodes, $|\Gamma(S)| \geq (1 - \epsilon)M$. (Here $\Gamma(S)$ denotes the neighbors of $S$.) Show that $(N, M, D = \lceil \log_2 N \rceil, K = M, 1/2)$-dispersers exist for all integers $N \geq M \geq 2$. 