

1. Jukna, Problem 25.5.
2. Recall that a (v, k, λ) -*nonstandard design* is a collection of subsets of $[v]$ such that
 - (a) each subset has exactly k points;
 - (b) each pair of subsets intersects in *at most* λ points.

For any prime p , give an explicit (p^2, p, d) -nonstandard design containing p^{d+1} subsets.
Hint: use polynomials.

3. An $(n, k, d)_q$ code is *perfect* if, for every $v \in \mathbb{F}_q^n$, there is a unique codeword w such that $\Delta(v, w) < d/2$. Show that any $(2^\ell - 1, 2^\ell - \ell - 1, 3)$ binary code is perfect.
4. Suppose you have an efficient program P which purportedly computes some unknown linear function $f : \mathbb{F}_2^n \rightarrow \mathbb{F}_2^n$. The program P may be faulty, but it is correct on at least $4/5$ of the inputs. Give an efficient randomized algorithm to compute f which, for *each* input, is correct with probability at least $.9$.
Hint: first get it correct with probability at least $3/5$.
5. Consider the problem of deciding whether two integer *multisets* S and T are identical, i.e., each integer occurs the same number of times in both sets. This problem can be solved by sorting using $O(n \log n)$ operations, where $|S| = |T| = n$. Give a randomized algorithm to solve this using $O(n)$ arithmetic operations.
Hint: use polynomials.