
2. Problem 11.2.

3. Problem 11.7.

4. This problem shows that amplification by random walks on expanders is tight up to constant factors. Suppose there were a way to amplify the success probability of any RP algorithm as follows. If the RP algorithm uses $r$ random bits to achieve error at most $1/2$, then there is a new RP algorithm for the same problem which uses $r + \lceil k/2 \rceil$ random bits to achieve error at most $2^{-k}$, for any $k \leq 10r$. Conclude that RP = P.

5. Let $G$ be a $d$-regular graph on $n$ vertices with adjacency matrix $A$. Let the smallest eigenvalue of $A$ be $\lambda_n$ (so $\lambda_n < 0$).
   a) Show that for any set $S$ of size $s$, the number of edges with both endpoints in $S$ is at least
      \[
      \frac{1}{2} \left( \frac{ds^2}{n} + \lambda_n s \left( 1 - \frac{s}{n} \right) \right).
      \]
   b) Conclude that the size of the largest independent set is at most $-n\lambda_n/(d - \lambda_n)$. 