1. Problem 6.5.

2. Problem 6.12. Hint: removing edges never increases the effective resistance. Also, you need not construct a $d$-regular counterexample, but each degree should be either $d$ or $d + 1$.

3. You will need the following proposition involving a stopping time for a random walk \{X_t\} on a graph $G = (V, E)$ with stationary distribution \{\pi_v, v \in V\}. Recall that a stopping time is a random time which can be specified by an online algorithm, i.e., it doesn’t depend on the future.

**Proposition.** Let $i$ be a vertex and $S$ a stopping time such that $X_S = i$ and $E_iS$ is finite. Then $E_i$[number of visits to vertex $j$ before time $S$] = $\pi_jE_iS$.

Note that this implies the hitting time $h_{ii} = 1/\pi_i$.

a) Show that for $j \neq i$, $E_i$[number of visits to vertex $j$ before returning to $i$] = $\pi_j/\pi_i$.

b) Show that for $j \neq i$, $E_j$[number of visits to vertex $j$ before visiting $i$] = $\pi_jC_{ij}$, where $C_{ij}$ denotes the commute time between $i$ and $j$.

c) Conclude that for $j \neq i$, $\Pr_i$[visit vertex $j$ before returning to $i$] = $1/(\pi_iC_{ij})$.

4. Give the best upper bound you can on the expected time for a random walk on the $n$-cycle to visit each vertex $n$ times. You get full credit for showing $O(n^2 \log n)$, plus bonus points for better bounds. The correct answer is $\Theta(n^2)$. 