CS388R: Randomized Algorithms | Problem Set No. 7
David Zuckerman | due: Nov. 20, 2008

1. Problem 6.5.

2. Problem 6.12. Hint: removing edges never increases the effective resistance. Also, you need not construct a d-regular counterexample, but each degree should be either d or $d+1$.

3. You will need the following proposition involving a stopping time for a random walk $\{X_t\}$ on a graph $G = (V, E)$ with stationary distribution $\{\pi_v, v \in V\}$. Recall that a stopping time is a random time which can be specified by an online algorithm, i.e., it doesn't depend on the future.

   Proposition. Let i be a vertex and S a stopping time such that $X_S = i$ and $E_i S$ is finite. Then E_i[number of visits to vertex j before time S] $= \pi_j E_i S$.

   Note that this implies the hitting time $h_{ii} = 1/\pi_i$.

   a) Show that for $j \neq i$, E_i[number of visits to vertex j before returning to i] $= \pi_j/\pi_i$.

   b) Show that for $j \neq i$, E_j[number of visits to vertex j before visiting i] $= \pi_j C_{ij}$, where C_{ij} denotes the commute time between i and j.

   c) Conclude that for $j \neq i$, $\Pr_i$[visit vertex j before returning to i] $= 1/(\pi_i C_{ij})$.

4. Give the best upper bound you can on the expected time for a random walk on the n-cycle to visit each vertex n times. You get full credit for showing $O(n^2 \log n)$, plus bonus points for better bounds. The correct answer is $\Theta(n^2)$.