CS 302 Computer Fluency
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Boolean Logic

1. Experiment with Boolean queries on eBay. Give an example of a case where you had to use at least one instance of all three of the connectors NOT, AND and OR to get what you were looking for.

2. Give an example of a Google query where you had to use more than just the AND of keywords to narrow your search and find what you were looking for.

3. In class, I gave the Longhorns and Aggies example. Come up with a nontrivial Boolean expression to describe something you find interesting. By "nontrivial" I mean that your expression needs to use at least three different Boolen operators.
   a. Write your expression using meaningful words.
   b. Rewrite it using one-character symbols so that it is easy to work with.
   c. Show its truth table.

4. Construct truth tables for each of the following formulas:
   a. $\neg P \rightarrow R$
   b. $(P \wedge \neg R) \vee (S \wedge R)$

5. Use truth tables to prove each of the following Boolean identities. Doing this is easy. Construct the truth table for the left hand side. Then construct the truth table for the right hand side. If the final column of both tables is the same then the two expressions have the same truth values. In that case, we say that they are equivalent.
   a. $A \vee B = B \vee A$
   b. $\neg(A \vee B) = \neg A \wedge \neg B$ (one of De Morgan's laws)

6. Suppose that we already know:

   $(P \wedge Q) \rightarrow R$

   Then we are told:

   $\neg R$

   Can we believe this? In other words, is the following Boolean expression satisfiable:

   $((P \wedge Q) \rightarrow R) \wedge \neg R$

7. Suppose that we wanted to construct the truth table that corresponds to the following Boolean formula:

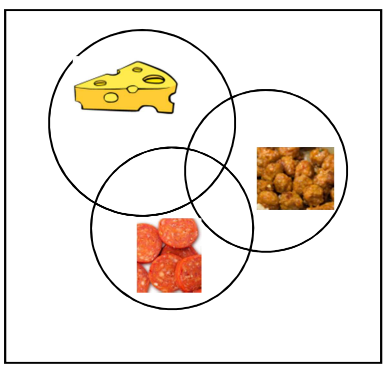
   $((P \vee S) \rightarrow R) \ \vee \ \neg(T \wedge A) \rightarrow B$

   How many rows would it contain?

8. In this problem, we will explore the similarities between set operations and Boolean ones. Define:
   - The intersection of two sets A and B, usually written $A \cap B$, is the set of objects that belong to both A AND B.
   - The union of two sets A and B, usually written $A \cup B$ (think U for union), is the set of objects that belong to A OR B (or possibly both).
   - The complement of a set A, usually written $\neg A$ (or sometimes ~A or !A) is the set of objects that do NOT belong to A.

   Venn diagrams, such as the ones shown in the first few slides of the Boolean logic lecture, are useful ways to visualize these set relationships.

   Consider the following Venn diagram:



For each of the following set relationships, draw a copy of the Venn diagram and color in the region(s) corresponding to the given set description:

a. Meat, defined as Pepperoni $\cup$ Sausage.
b. Vegetarian, defined as $\neg$Meat.
c. Vegan, defined as $\neg$Meat $\cap$ $\neg$Cheese. (Hint: Do this by hatching the $\neg$Meat region. Now hatch the $\neg$Cheese region in a different direction. Finally, you want the area that is in both hatched regions. Mark it clearly)
d. Lactose intolerant, defined as $\neg$Cheese.
e. Pop Those Arteries, defined as $\neg$Vegan. First mark this one. Now give a different definition of this same set. Use the $\cup$

9. Show the behavior of the following circuit with a truth table (in other words, show what it will output for each possible pair of A,B inputs). Describe the possible input values as 0 (False) and 1 (True).