CS 313k - Homework #2
100 points possible
Due: Monday, September 21, at the beginning of class

Give clear, legible answers to all questions. Staple the pages of your solution set together, and put your name and EID at the top of the first page.

1. For the following argument, assign propositional variables to represent the statements and write down the argument form. Then use a step by step proof, as we've done in class, to show that the argument is valid. Give a reason for each step in your proof. At the top of your proof (and all your validity proofs), you should have this header:

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   Step  Claim     Reason
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   Argument:
   It is not raining this afternoon and it is warmer than yesterday.
   We will go to the movie only if it is raining.
   If we do not go to the movie, then we will go to the park.
   If we go to the park, then we will be home early.
   Therefore we will be home early.

2. As in the first question, do a step by step proof to show the following argument is valid:
   $(P \wedge Q) \vee R$
   $\underline{R \rightarrow S}$
   $\therefore P \vee S$

3. Is the following argument valid? Either prove that it is valid, or give a counterexample to show it is not valid.
   If it does not rain or it is not foggy, then we will sail our boat today and we will have a picnic.
   If we sail the boat, then the deck will be cleaned.
   The deck was not cleaned.
   Therefore it rained.

4. Show that $(P \wedge Q) \rightarrow R$ and $(P \rightarrow R) \wedge (Q \rightarrow R)$ are not logically equivalent.

5. Let predicate P(x,y) be "x is the capitol of y". Write down the following propositions in English, and determine the truth value of each.

   (a) P(Birmingham, Alabama)

   (b) P(Omaha, Nebraska)

   (c) P(Denver, Colorado)

6. Translate the following quantified statements into English.
   P(x) is "x studies more than 3 hours a day", R(x) is "x likes to make good grades" and the universe is the set of all students.

   (a) $\exists x P(x)$

   (b) $\forall x P(x)$

   (c) $\forall x(P(x) \rightarrow R(x))$

   (d) $\exists x(P(x) \vee R(x))$

   (e) $\neg \forall x P(x)$

7. For each of the following statements, find a domain for which the statement is true and a domain for which the statement is false.

   (a) Everyone is studying logic.

   (b) Everyone has the same mother.

   (c) $x \geq 1$

8. Translate the English statements into logic. Let P(x) be "x believes in Santa Claus", Q(x) is "x lives in Kentucky", and R(x) is "x has a cat". The domain is the set of all 3rd graders.

   (a) A student believes in Santa Claus or has a cat.

   (b) All 3rd graders believe in Santa Claus or have a cat.

   (c) All 3rd graders who live in Kentucky believe in Santa Claus.

   (d) No 3rd grader believes in Santa Claus.