

CS 313k - Homework #3  
100 points possible  
Due: September 28 at the beginning of class

Give clear, legible answers to all questions. Staple the pages of your solution set together, and put your name and EID at the top of the first page.

1. Translate the following statements into logical notation using predicates and quantifiers. The domain is the set of integers.
  - (a) The product of two negative integers is positive.
  - (b) The average of two positive integers is positive.
  - (c) The difference of two negative integers is not necessarily negative.
2. Let  $Q(x, y)$  be “ $x+y = x-y$ ”. The domain for  $x$  and  $y$  is  $\mathbb{Z}$ . Are the following true or false?
  - (a)  $Q(1,1)$
  - (b)  $\forall y Q(1, y)$
  - (c)  $\exists x \exists y Q(x, y)$
  - (d)  $\exists y \forall x Q(x, y)$
  - (e)  $\forall x \exists y Q(x, y)$
  - (f)  $\forall y \exists x Q(x, y)$
3. Write down the negation of each statement so that the negation appears directly in front of a predicate.
  - (a)  $\neg \exists y \exists x P(x, y)$
  - (b)  $\neg \exists y (Q(y) \wedge \forall x \neg T(x, y))$
  - (c)  $\neg \forall x \exists y P(x, y)$
  - (d)  $\neg \exists y (\forall x \exists z R(x, y, z) \vee \exists x \forall z S(x, y, z))$
4. Show the following statements are false by finding a counterexample for each statement. The domain for all variables is  $\mathbb{Z}$ .
  - (a)  $\forall x \exists y (x = \frac{1}{y})$
  - (b)  $\forall x \exists y (y^2 - x < 100)$
  - (c)  $\forall x \forall y (x^2 \neq y^3)$

5. What rule of inference is used in the following arguments?
- (a) Bears live in China and are mammals. Therefore bears are mammals.
  - (b) Lisa is a good rower. If Lisa is a good rower, then she can be on the rowing team. Therefore Lisa can be on the rowing team.
  - (c) Scott will work for IBM this summer. Therefore this summer Scott will work at IBM or he will lie on the beach every day.
6. Write down the argument form, and prove the argument is valid. Hara, an employee of company XYZ, owns a black miata. Everyone who owns a black miata has gotten at least one speeding ticket. Therefore someone who works for company XYZ has gotten a speeding ticket.
7. Determine if the argument is correct or not and prove your answer. (You must write the argument form before you give your proof).
- (a) Everyone who attends UT has lived in a dorm. Mike has never lived in a dorm. Therefore Mike does not attend UT.
  - (b) A miata is fun to drive. Vanessa's car is not a miata. Therefore Vanessa's car is not fun to drive.
  - (c) Oliver likes funny movies. Oliver likes the movie *Up and Down*. Therefore *Up and Down* is a funny movie.
8. Is the argument valid? Prove your answer.  
If  $x$  is a positive real number, then  $x^2$  is a positive real number. Therefore if  $c^2$  is positive and real, then  $c$  is a positive real number.