CS 313k - homework 7

Staple the pages of your solution set together, and put your name and EID on the top of the first page. Answer each question clearly. The logic you use to produce your answers is the most important thing.

1. For each of the following binary relations, clearly state whether or not the relation is an equivalence relation, and prove your answer.

   (a) Let R be the relation on $\mathbb{Z}$ given by:
   $xRy \leftrightarrow x + y$ is a multiple of 3.

   (b) Let R be the relation on $\mathbb{Z}$ defined by:
   $xRy \leftrightarrow x + y < 5$.

   (c) Let R be the relation on $\mathbb{N}$ defined by:
   $xRy \leftrightarrow xy$ is even.

2. A binary relation R is **irreflexive** if $\forall x[(x, x) \notin R]$. Give an example of a relation that is neither reflexive nor irreflexive. Explain why it is not reflexive or irreflexive. You must define the relation R as well as the set A on which R is defined.

3. Prove or disprove: For any reflexive relations R and S on set A, $R - S$ is irreflexive.

4. Let $A = \{1, 2, 3, 4, 5, 6\}$, and consider the sets $A_1 = \{1\}, A_2 = \{3, 5, 6\}, A_3 = \{2, 4\}$. Note that $\{A_1, A_2, A_3\}$ is a partition of A. Write down the equivalence relation R on A that is defined by this partition.

5. Let R be the relation on $2^{\mathbb{Z}} - \{\emptyset\}$ defined by:
   $ARB \leftrightarrow A \cap B \neq \emptyset$.

   (a) Prove or disprove: R is reflexive.

   (b) Prove or disprove: R is symmetric.

   (c) Prove or disprove: R is transitive.

   (d) Prove or disprove: R is anti-symmetric.

6. Let R be the relation on $2^{\{a,b,c,d\}}$ defined by:
   $ARB \leftrightarrow |A| = |B|$. Prove that R is an equivalence relation, and list the distinct equivalence classes of R.

7. Let $A = \{2, 3, 4, 6, 9, 27, 36\}$ and define relation R on A such that $aRb \leftrightarrow a|b$. Prove that R is a poset. Then

   (a) Find the maximal elements of (A, R).

   (b) Find the minimal elements of (A, R).

(c) Is there a greatest element? If so, what is it?

(d) Is there a least element? If so, what is it?

8. Define a poset (A, R) that has a minimal element but no maximal element.

9. Let A be the set of all bit strings, and let $B = \mathbb{Z}$. Define function $f : A \to B$ by $f(b) = |b|$. Give a proof or a counterexample to justify your answers for the following:

   (a) Is f 1-1?

   (b) Is f onto B?

10. section 6.1, 2(p.265)