

CS341 Automata Theory

Final Exam Review Sheet

1. For each of the following languages, determine if the language is (I) regular, (II) context-free but not regular, (III) not context-free. Prove your answer.
 - (a) $L = \{a^n b^{n+2} \mid n \geq 0\}$
 - (b) $L = \{xay \mid x, y \in \{a, b\}^+\}$
 - (c) $L = \{0^n 1^{m+n} 2^{m+n} \mid m, n \geq 0\}$
2. Give both a CFG and a PDA for the following languages.
 - (a) $L = \{a^k c^3 b^r \mid r > k\}$
 - (b) $L = \{w \in \{0, 1\}^* \mid w = w^R\}$
 - (c) $L = \{w \in \{a, b\}^* \mid |w| \text{ is odd and the middle symbol of } w \text{ is a}\}$
3. Give a regular expression and a FA for the following languages.
 - (a) $L = \{w \in \{0, 1\}^* \mid w \text{ begins with a 1 and ends with a 0}\}$
 - (b) $L = \{w \in \{0, 1\}^* \mid w \text{ contains substrings 010 and 101}\}$
4. Give the state diagram of a recognizer for the following.
 - (a) $L = \{a^i b^i c^j \mid i, j \geq 0, j > i\}$
 - (b) $L = \{0^n 1^{2n+1} \mid n \geq 0\}$
5. Define a TM that computes the function f where $f(x) = 2x + 1$.
6. Prove that the decidable languages are closed under complement.
7. Prove that $L = \{\langle M \rangle \mid M \text{ is a TM and } M \text{ accepts all strings of length less than 5 in 100 steps or less}\}$ is decidable.
8. Assuming only that A_{TM} is not decidable, prove that $HALT_{TM}$ is not decidable.
9. Prove that $\{\langle M \rangle \mid M \text{ is a TM and if } M \text{ accepts any string } w, \text{ then } M \text{ also accepts } w^r\}$ is not decidable.
10. Prove the following languages are SD but not decidable:
 - (a) $L = \{\langle M \rangle \mid M \text{ is a TM and } M \text{ accepts at least 2 strings}\}$
 - (b) $L = \{\langle M_1, M_2 \rangle \mid M_1, M_2 \text{ are TMs and } aa \in L(M_1) \cup L(M_2)\}$